Higgs theory, radiative corrections and current developments

Johannes Braathen (DESY)

7th International Workshop on "Higgs as a Probe of New Physics 2025"

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

CLUSTER OF EXCELLENCE QUANTUM UNIVERSE

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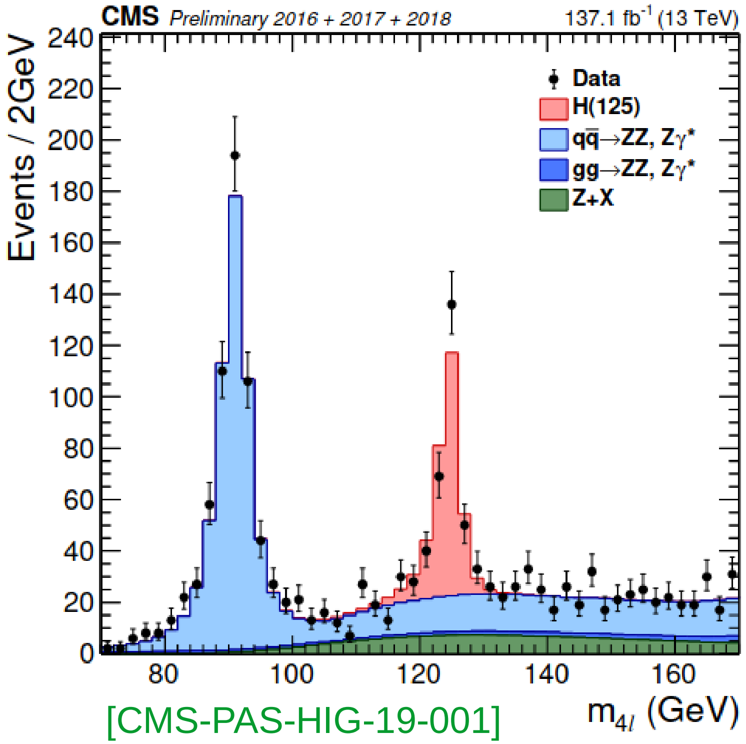
Higgs discovery in 2012: a milestone for Particle Physics

- *4th July 2012*: discovery of **a** **Higgs boson** of mass 125 GeV by ATLAS and CMS collaborations at CERN Large Hadron Collider was a major milestone for Particle Physics



[CMS-PAS-HIG-19-001]

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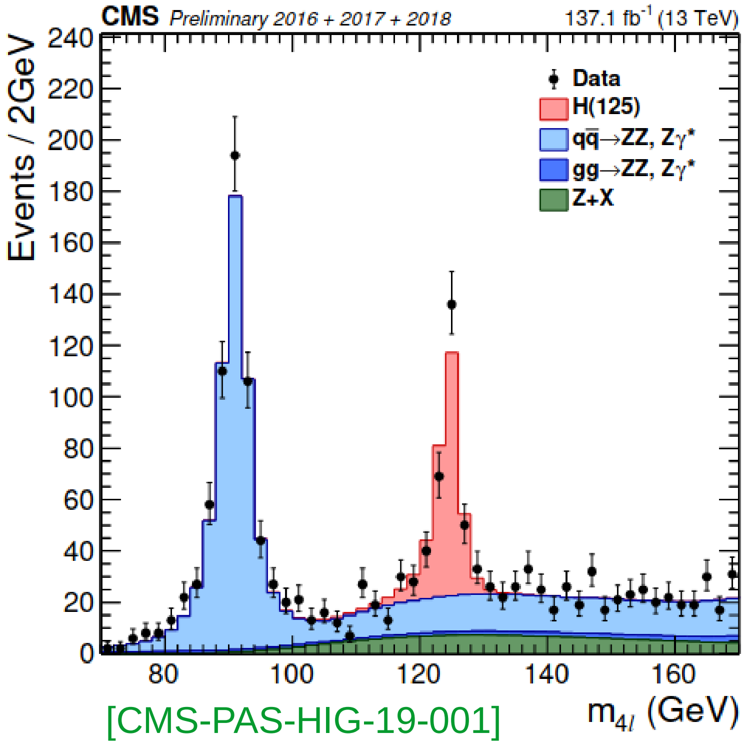
- Its mass **M_h=125 GeV**, to astonishing ~0.1% precision!
- The electroweak (EW) vacuum expectation value (vev) **v=246 GeV**
- Spin 0
- Not purely CP-odd



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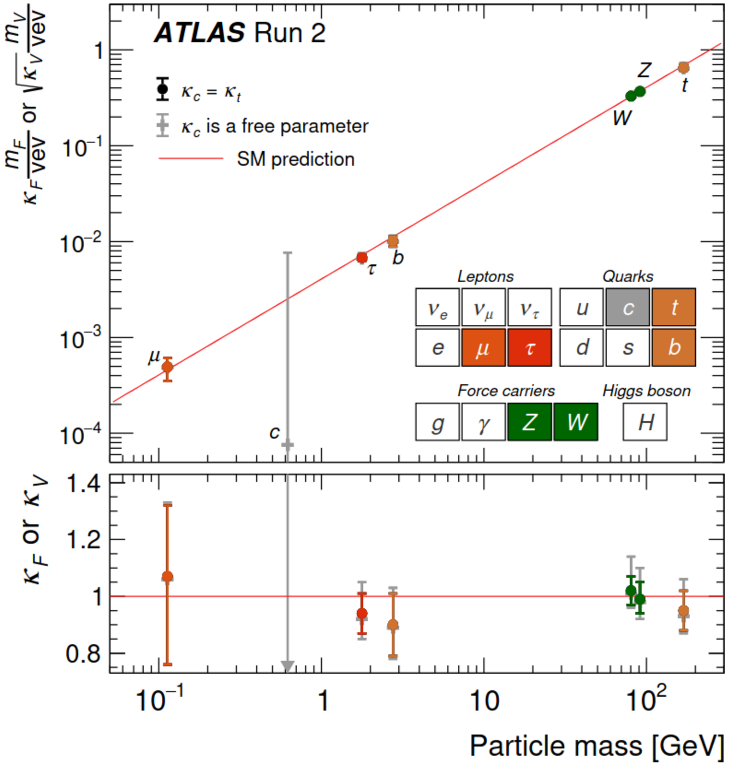

[ATLAS CERN-EP-2022-057]

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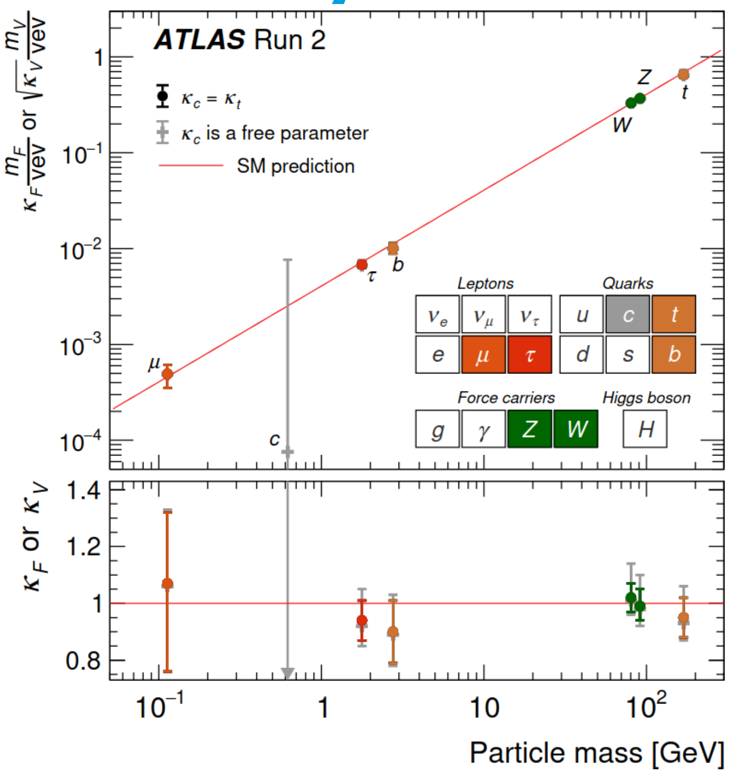
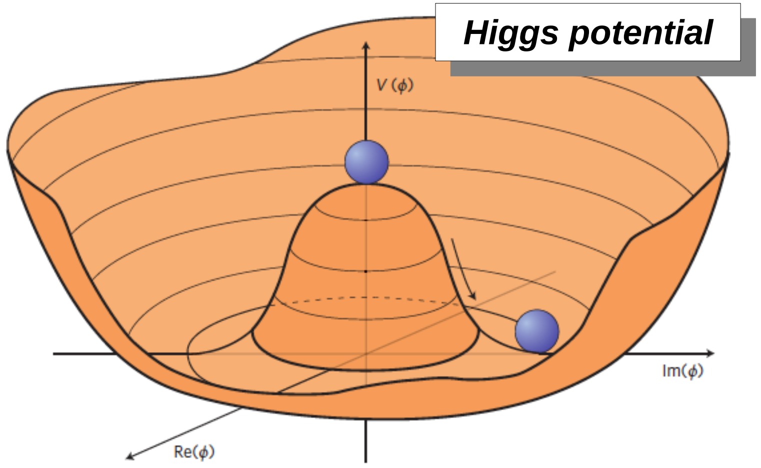


[ATLAS CERN-EP-2022-057]

Higgs potential



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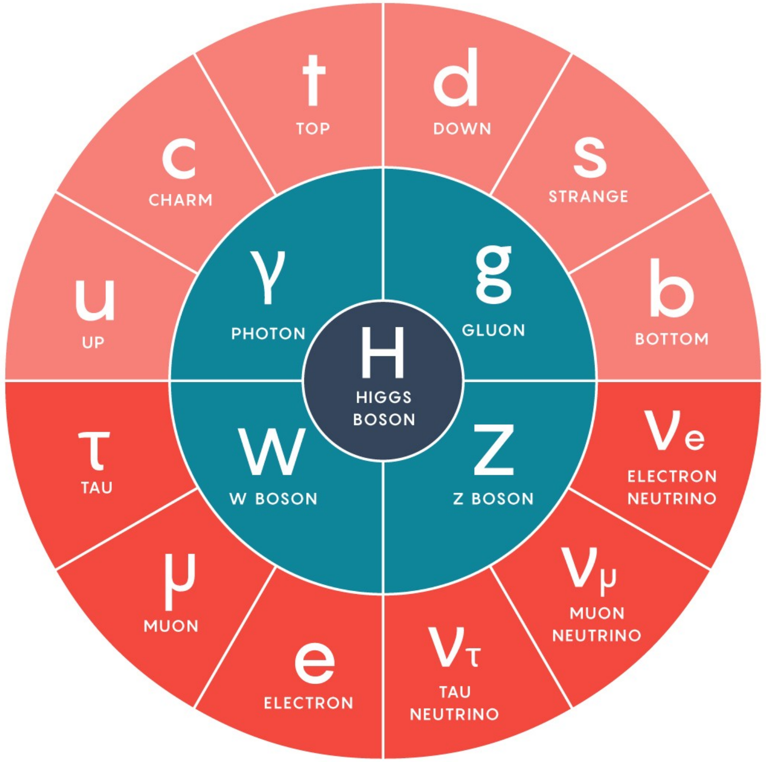

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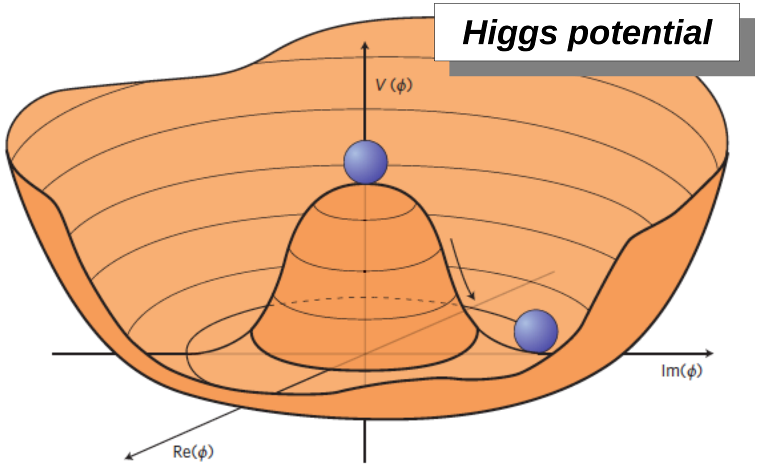

➢ Particle content of **Standard Model** is "complete"
→ *is this the end of the story?*

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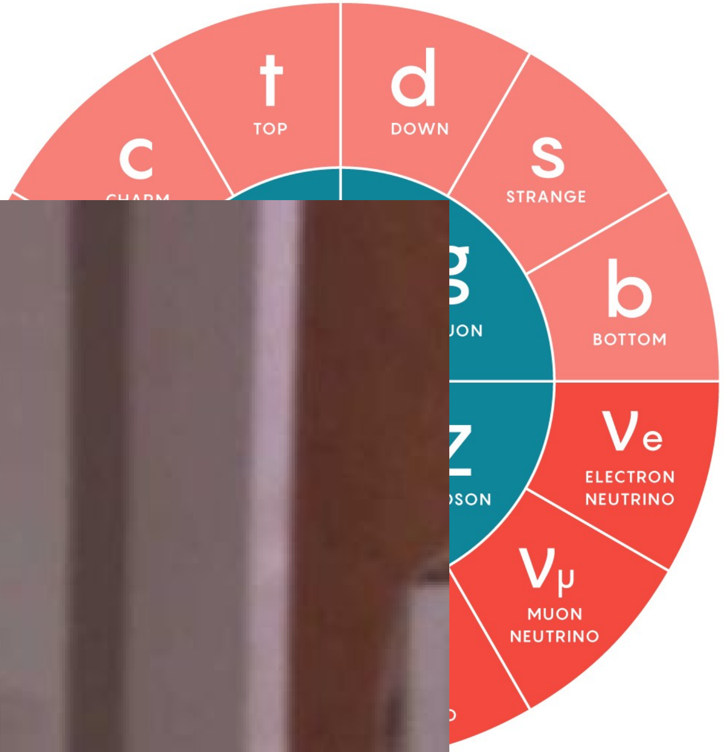
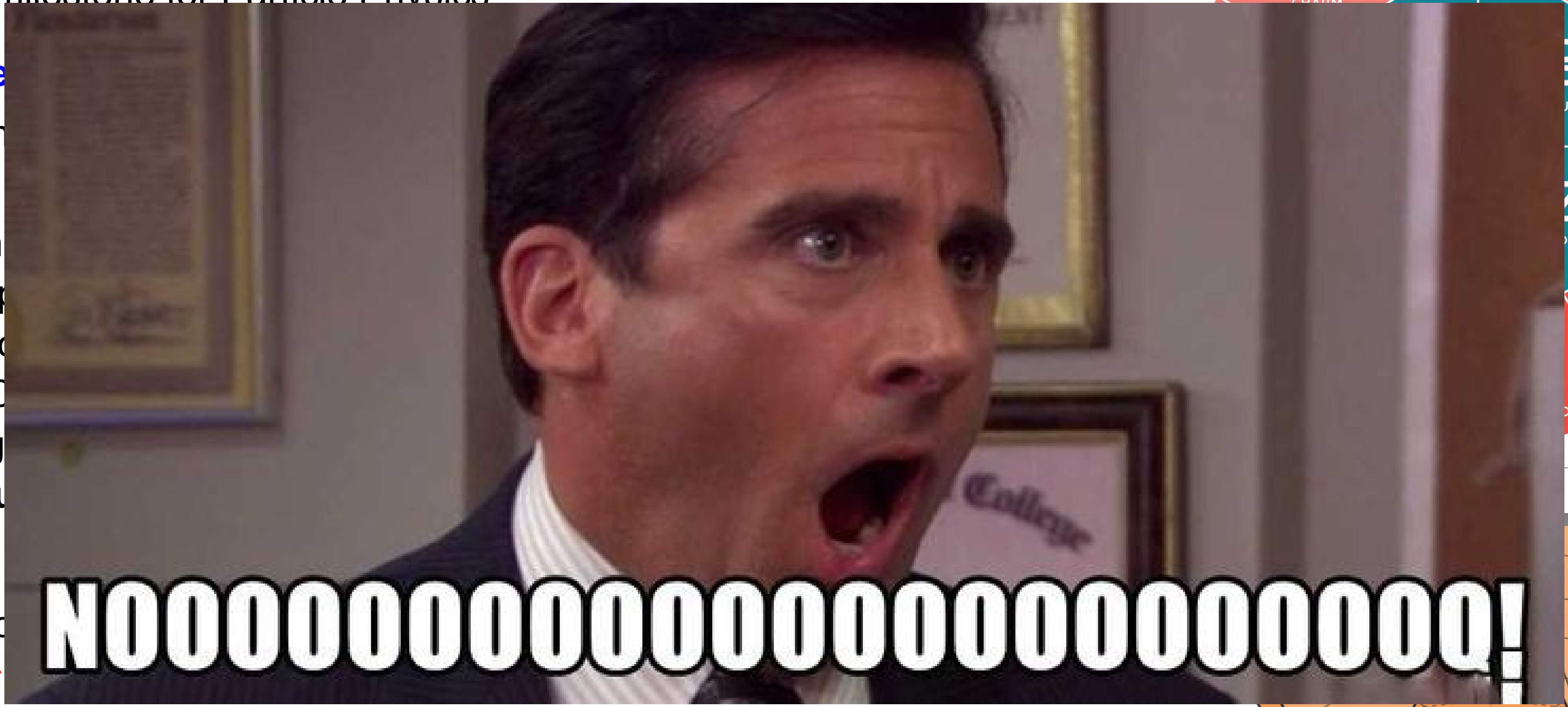
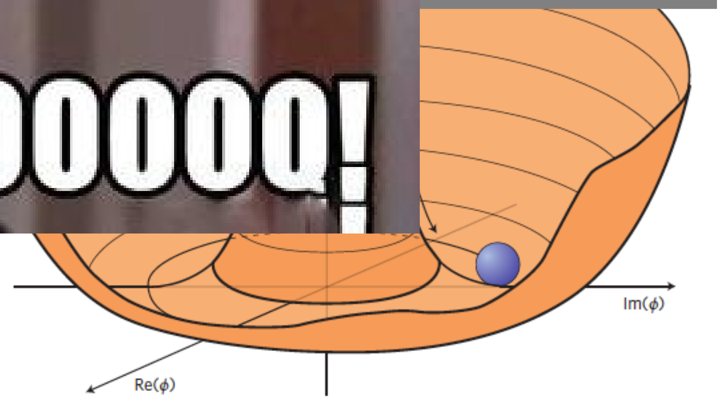
➢ Particle c
→ *is this*





Higgs potential



Open questions and motivation for New Physics

- **What we still don't know about the Higgs boson**:
  - Its couplings to 1st and 2nd gen. fermions
  - Its total width; BR(h→inv.) < 9%
  - Its CP nature
  - Its fundamental nature (elementary or composite)
  - Structure of the Higgs sector (minimal or extended)
  - Form and origin of the Higgs potential (i.e. *why* do particles get masses, not just *how*)

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- **Many further questions remain unanswered**, e.g.
  - Gauge hierarchy problem, i.e. why is gravity so much weaker than the other forces (or why is the Planck scale so much higher than the electroweak scale)
  - Reason for 3 fermion families and origin of flavour
  - Origin of matter-antimatter asymmetry of the Universe
  - Dark Matter

  Etc.

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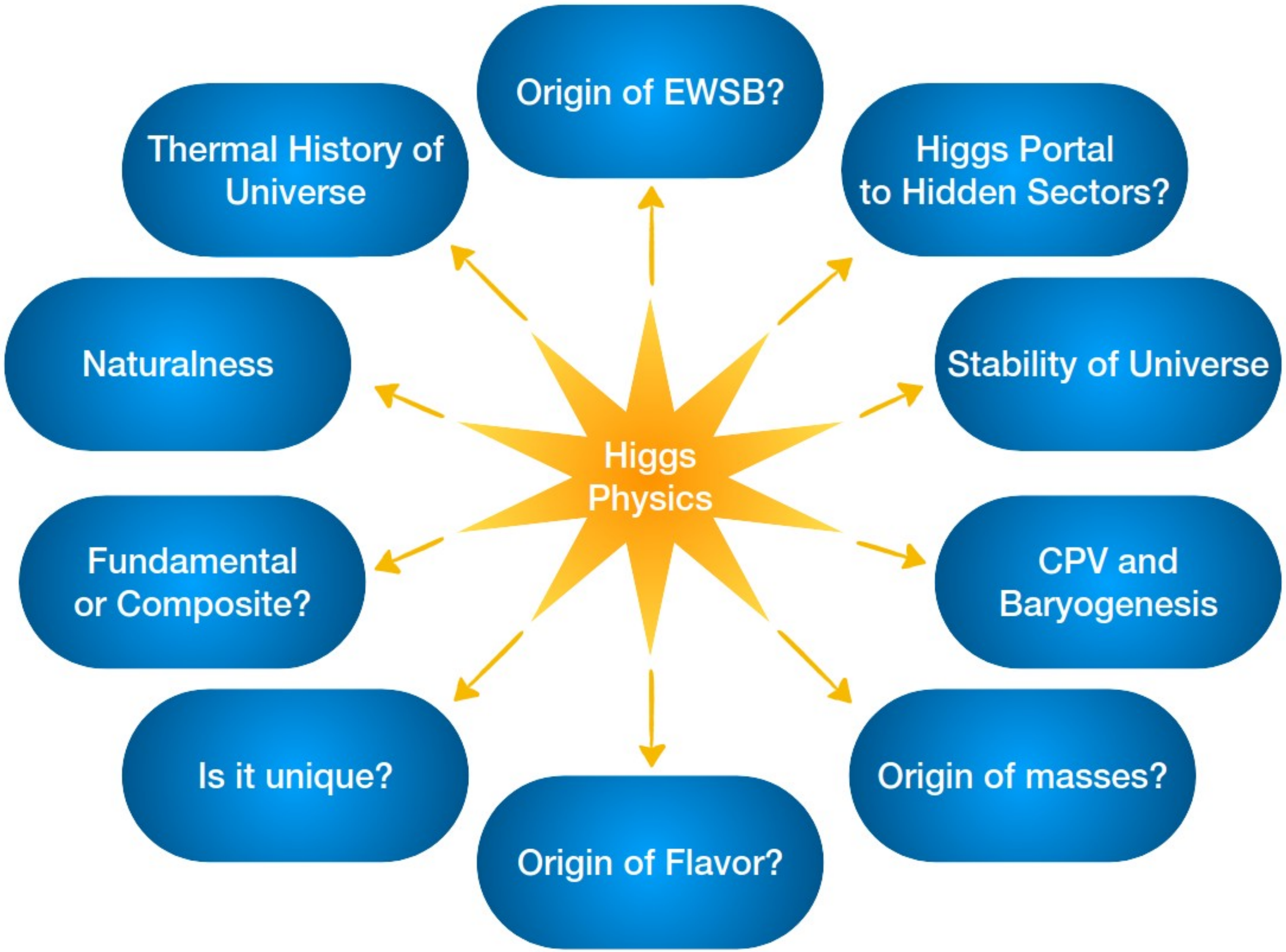
- Not addressed by our current best description of Particle Physics, the Standard Model (SM)
  → **New Physics must exist beyond-the-Standard-Model (BSM)!**

- Many open problems relate to Higgs sector
  → the 125-GeV Higgs boson will certainly play a **key role in understanding the nature of BSM Physics**
  → BSM models often feature **additional Higgs bosons/scalars**

The Higgs boson plays a central role to probe New Physics



[Snowmass report on Higgs Physics 2209.07510]

Outline of the lectures

- **Part 1: Higgs as a Probe of New Physics**

- **Part 2: Some basics on radiative corrections and theory uncertainties**

- **Part 3: Higgs measurements and precision calculations**

- **Part 4: Di-Higgs production – theory calculations, uncertainties and current developments**

- **Part 5: Future prospects (brief selection)**

Probing New Physics with the Higgs boson

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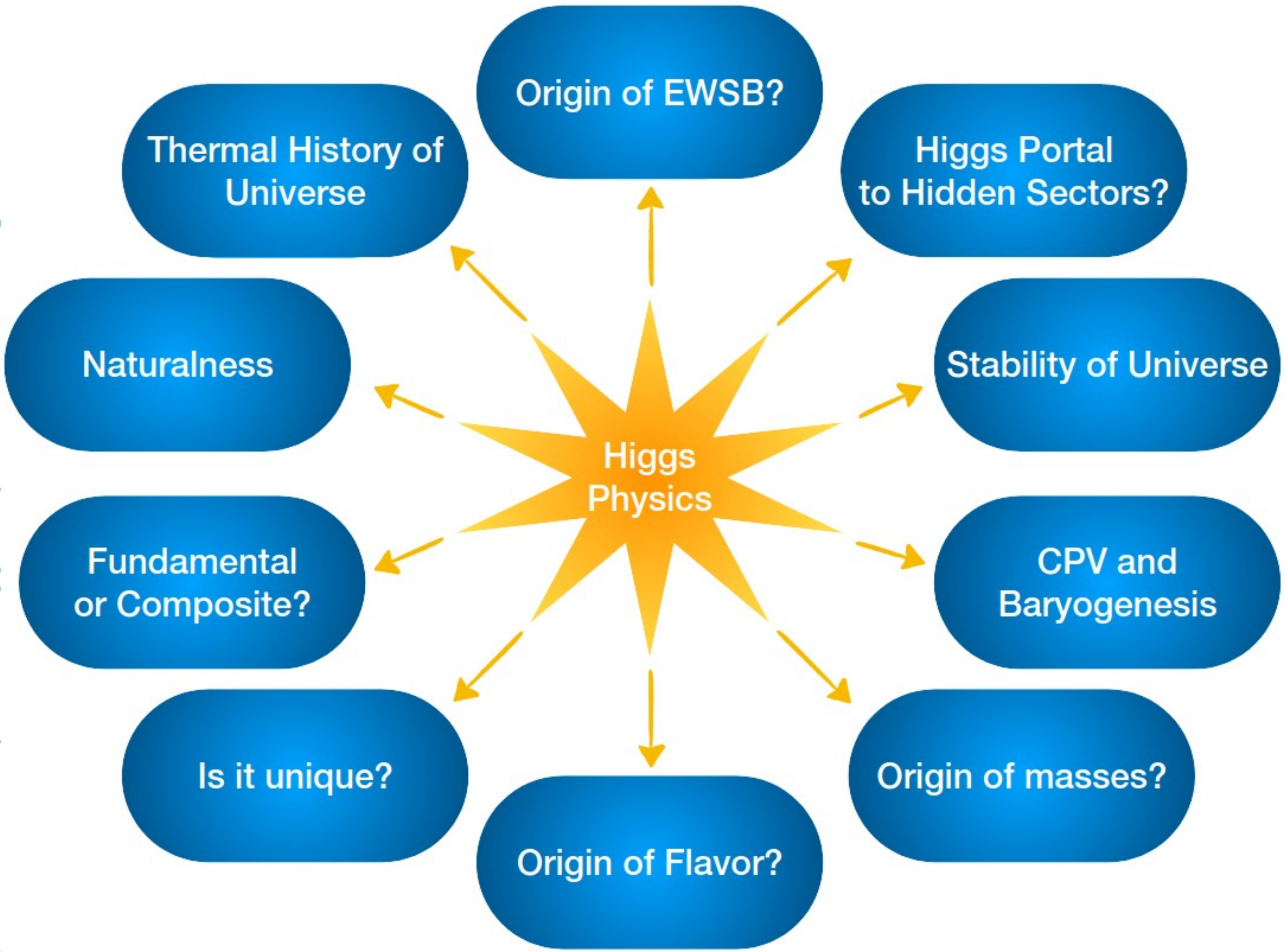
[Snowmass report on Higgs Physics 2209.07510]



Overview of this part:
- hierarchy problems
- form of the Higgs potential
- baryogenesis and electroweak phase transition

In backup:
- more details on hierarchy problems and their solutions
- Higgs portal to dark sectors
- Higgs as inflaton
- neutrino mass models with extended Higgs sectors

Hierarchy problems in Higgs Physics

Slide adapted from [Salam ‘23], itself adapted from [Giudice]

Parts of the SM Lagrangian involving only gauge bosons and fermions:

$$\mathcal{L} \supset -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \bar{\psi}_i \gamma_\mu D^\mu_{ij} \psi_j$$

→ entirely constrained by gauge symmetry, tested to high precision (e.g. at LEP)

Parts of the SM Lagrangian involving the Higgs field:

$$\mathcal{L} \supset -y_{ij} \bar{\psi}_i \Phi \psi_j + \mu^2 |\Phi|^2 + \lambda |\Phi|^4 - V_0$$

Yukawa couplings: Hierarchy of fermion masses and flavour

Higgs mass term: Gauge hierarchy problem

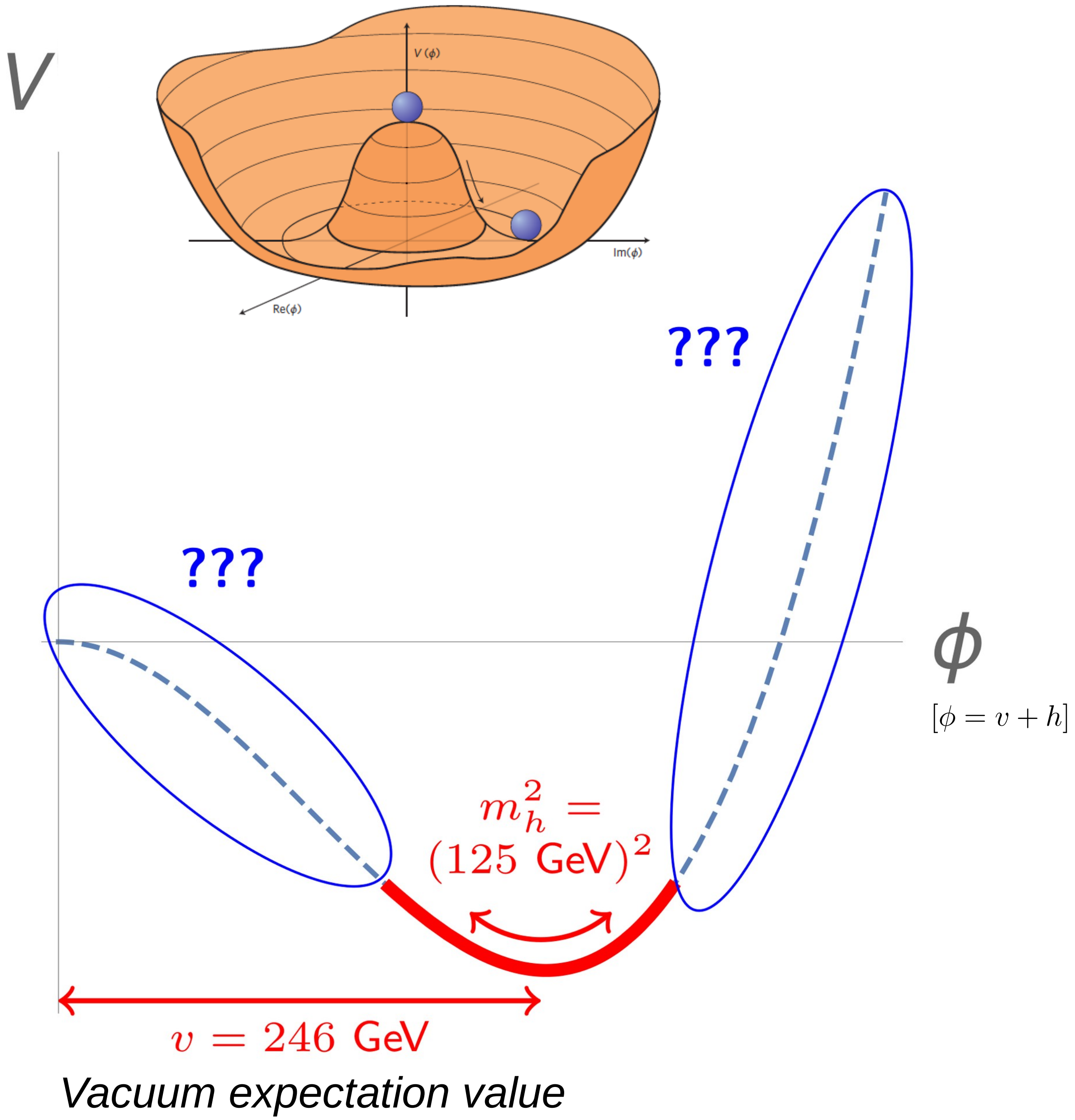
Quartic Higgs coupling: UV behaviour and vacuum stability

Vacuum energy: Cosmological constant problem

[More details on hierarchy problems, and on possible solutions to gauge hierarchy problem in backup]

Form of the Higgs potential and trilinear Higgs coupling

- Brout-Englert-Higgs mechanism = **origin of electroweak symmetry breaking** ...
  … but very little known about the **Higgs potential** causing the phase transition



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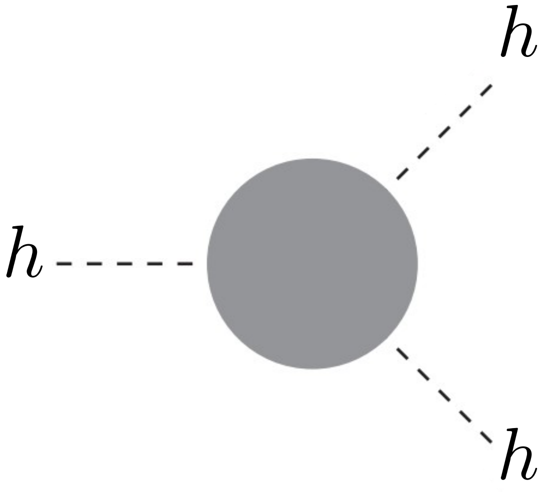
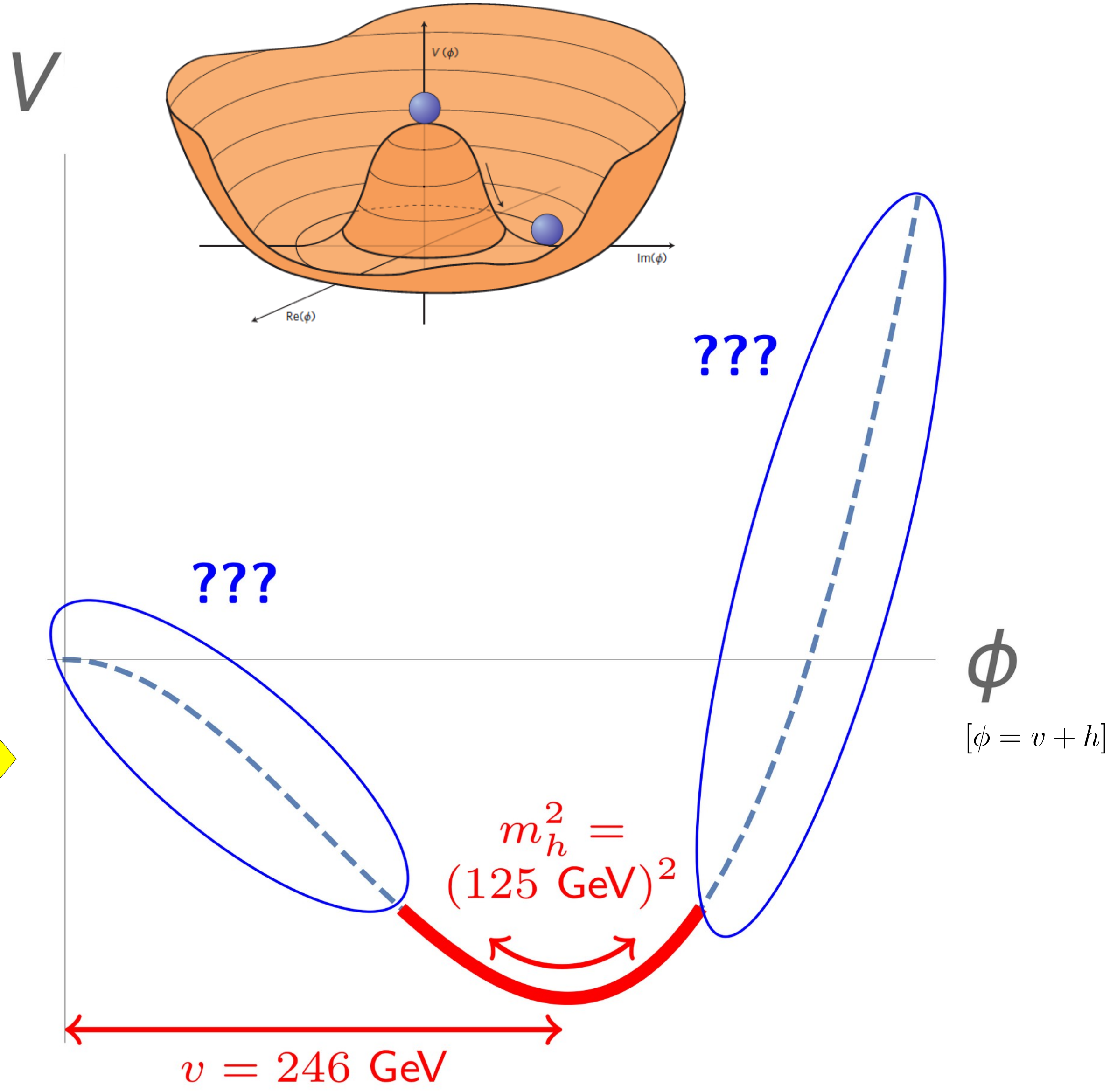
- **Trilinear Higgs coupling λ_{hhh}** crucial to understand the shape of the potential





Vacuum expectation value

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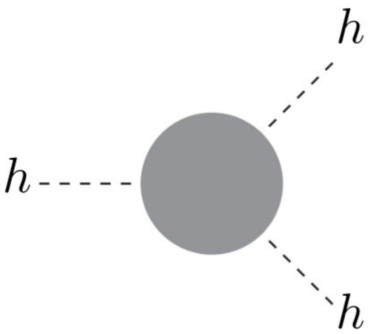

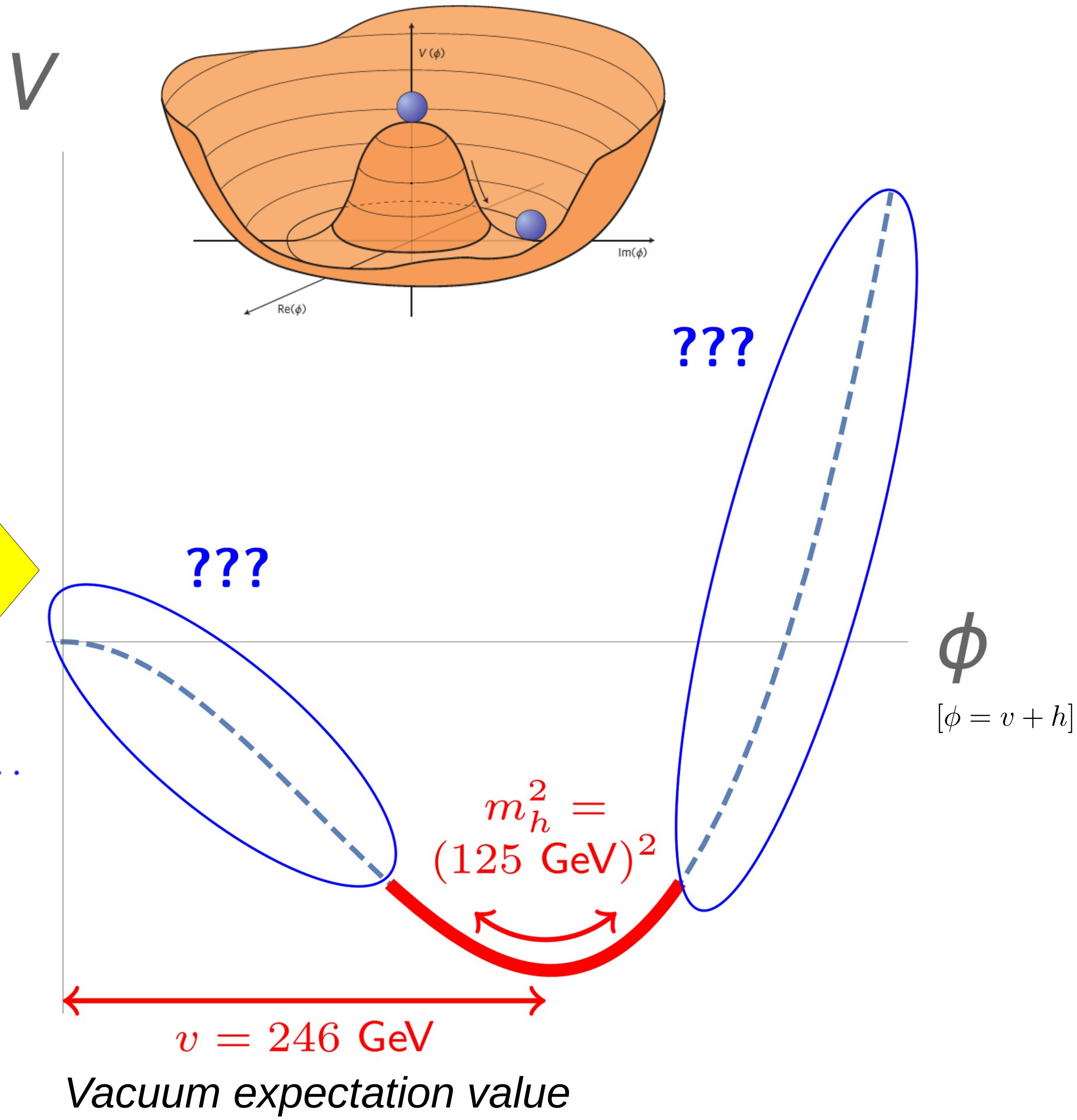

In the SM: $$V_{\rm SM}^{(0)} = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \underbrace{\left(\frac{3m_h^2}{v}\right)}_{\equiv(\lambda_{hhh}^{(0)})^{\rm SM}} h^3 + \frac{1}{4!} \underbrace{\left(\frac{3m_h^2}{v^2}\right)}_{\equiv(\lambda_{hhhh}^{(0)})^{\rm SM}} h^4 + \cdots$$

In general:
$$V^{(0)} = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \overbrace{\kappa_\lambda \left(\frac{3m_h^2}{v}\right)}^{\equiv\lambda_{hhh}} h^3 + \frac{1}{4!} \kappa_4 \left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$$
with $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{\rm SM}$ and $\kappa_4 \equiv \lambda_{hhhh}/(\lambda_{hhhh}^{(0)})^{\rm SM}$

Baryogenesis

- **Observed Baryon Asymmetry of the Universe (BAU)**

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$ [Planck ‘18]

n_b: baryon no. density
$n_{\bar{b}}$: antibaryon no. density
n_γ: photon no. density

- **Sakharov conditions** [Sakharov ‘67] for a theory to explain BAU:
  1) Baryon number violation
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In the SM

→ Sphaleron transitions (break B+L)
→ C violation (SM is chiral), but not enough CP violation
→ No loss of th. eq. → in SM, the EWPT is a crossover

SM phase diagram

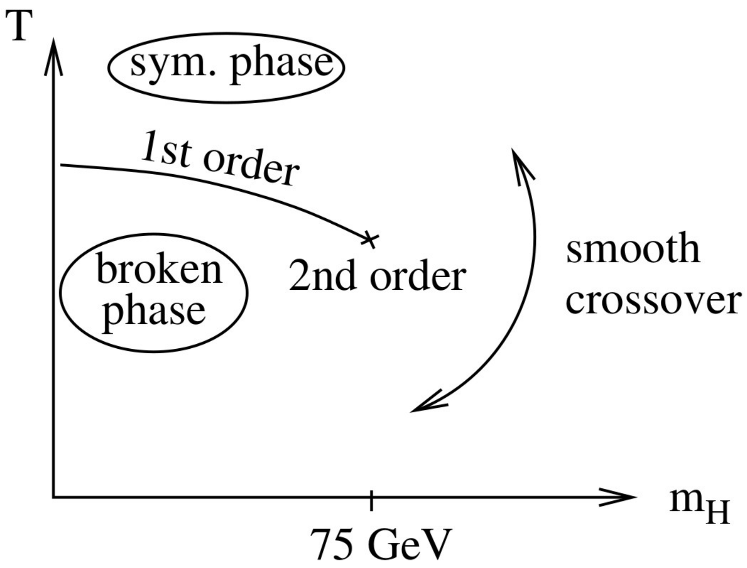


Figure from [Cline '06]

- SM cannot reproduce the BAU → **BSM physics needed!**

Electroweak Baryogenesis

- Many scenarios proposed, including:
  - Grand Unified Theories
  - Leptogenesis
  - **Electroweak Baryogenesis** (EWBG) [Kuzmin, Rubakov, Shaposhnikov, ‘85], [Cohen, Kaplan, Nelson ‘93]

- **Sakharov conditions** in EWBG

1) Baryon number violation → Sphaleron transitions (break B+L)

2) C and CP violation → C violation + CP violation in extended Higgs sector

3) Loss of thermal equilibrium → Loss of th. eq. via a strong 1st order EWPT

[More details on electroweak baryogenesis in backup]

The Higgs potential and the Electroweak Phase Transition

Possible thermal history of the Higgs potential:

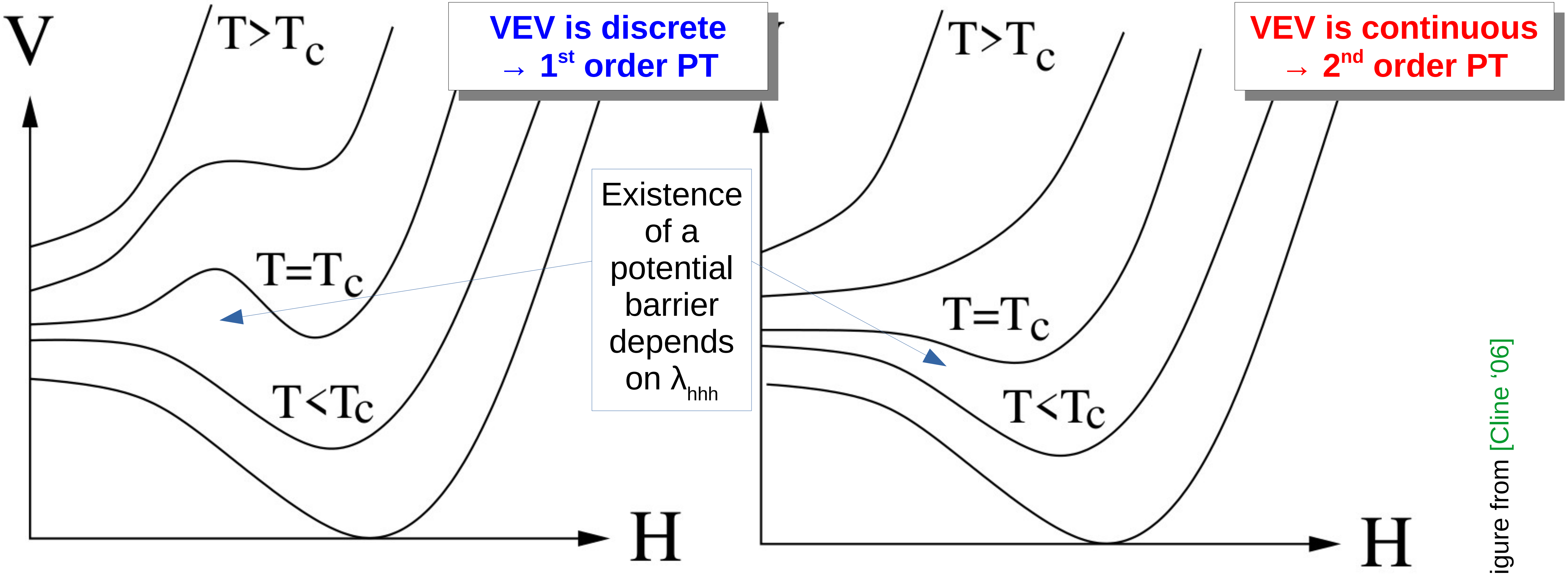
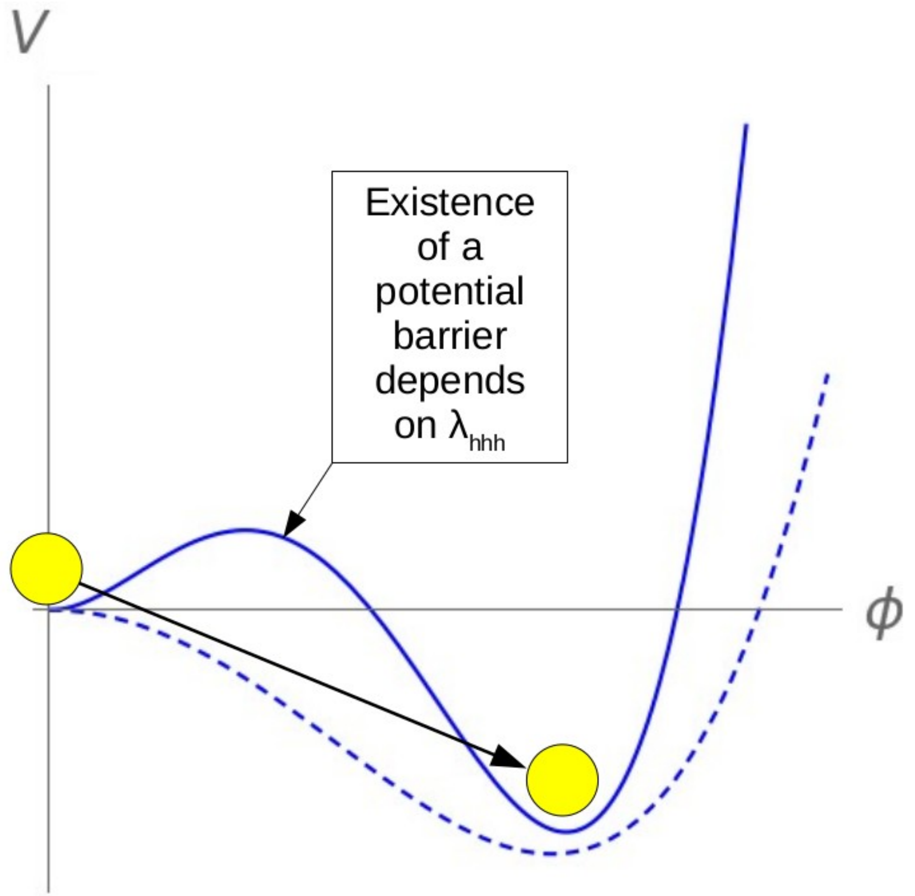


Figure from [Cline '06]

- **λ_{hhh} determines the nature of the EWPT!**

  ⇒ deviation of λ_{hhh} from its SM prediction typically* needed to have a strongly first-order EWPT [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

The Higgs potential and the Electroweak Phase Transition



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  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

Radiative corrections 101

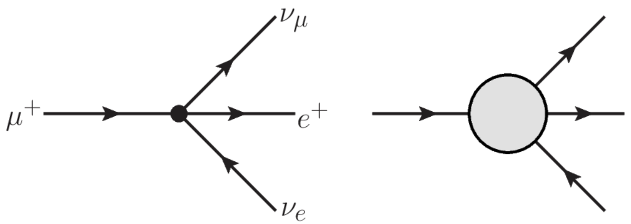
Impact of radiative corrections: *W*-boson mass

- Electroweak precision observables, such as
  - **W-boson mass M_W**
  - Effective leptonic weak mixing angle → $\sin^2\theta_{eff}^{lep}$
  - Z-boson decay width Γ_Z
  - Muon anomalous magnetic moment $(g-2)_\mu$

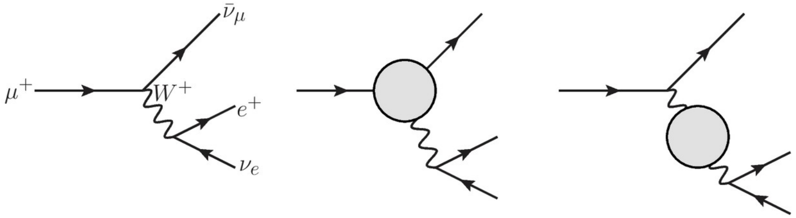
  etc.

  are **measured very precisely**, and can also be **computed to high level of accuracy** in terms of **G_F, $\alpha(0)$, M_Z** (most precisely measured EW quantities) and M_h, M_t, α_S , $\Delta\alpha^{had}$, $\Delta\alpha^{lept}$, m_b, etc.

Muon decay in Fermi theory:



Muon decay in SM:



- Relation between M_W, M_Z, G_F, $\alpha(0)$ obtained by matching the calculation of muon decay between the SM and Fermi theory

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

  with $\Delta r \equiv \Delta r(M_W, M_Z, M_h, M_t, \ldots)$ the corrections to muon decay (w/o finite QED effects)

- Without Δr (loop corrections), M_W ~ 80.9 GeV, i.e. **~40σ away** from experimental measurement! One-loop calculation also **~10σ off** → incorporation of (known) higher orders is essential
- **Allows testing the SM as well as BSM models at quantum level**

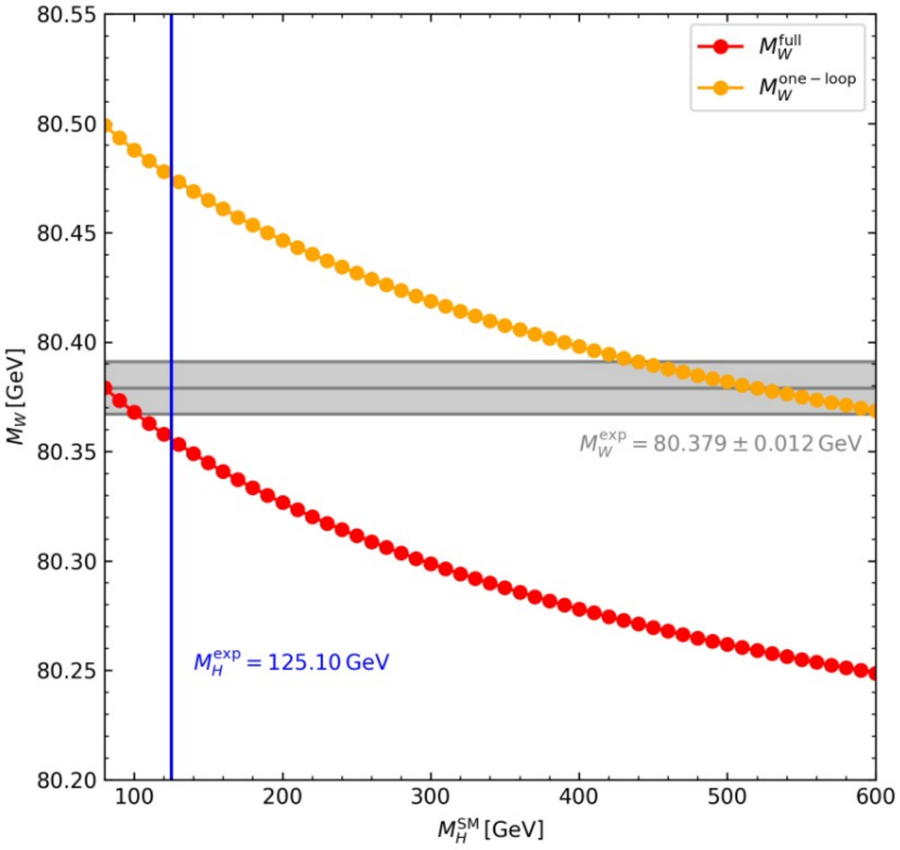
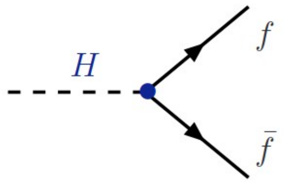
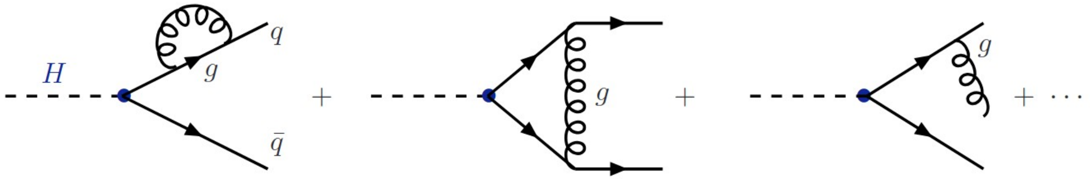


Figure by G. Weiglein

Impact of radiative corrections: h→b$\bar{b}$ decay



Left: LO
Bottom: NLO QCD



- **Decay h → bb already at tree level** (driven by bottom Yukawa coupling, prop. to m_b)

$$\Gamma^{(0)}(h \to b\bar{b}) \overset{M_h \gg m_b}{=} \frac{3G_F}{4\sqrt{2}\pi} M_h m_b^2 \qquad \textbf{(1)}$$

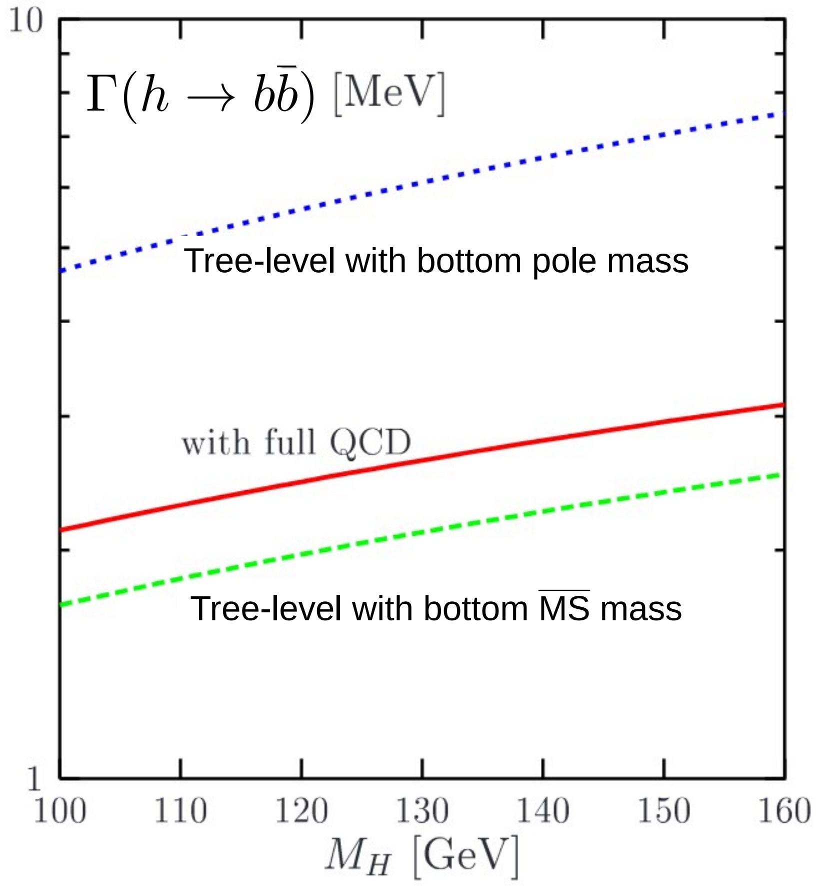
- Large QCD corrections (driven by strong gauge coupling α_s which does not enter at tree level)

$$\hat{\Gamma}(h \to b\bar{b}) \overset{M_h \gg m_b}{=} \frac{3G_F}{4\sqrt{2}\pi} M_h m_b^2 \left[1 + \frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{9}{4} + \frac{3}{2}\log\frac{m_b^2}{M_h^2}\right) + \cdots\right] \qquad \textbf{(2)}$$

also contains **large logs log(m_b/M_h)** → **can spoil perturbative expansion**

- Can be resummed to all orders in α_s (→ c.f. Effective Field Theories) by expressing the decay width in terms of **$\overline{\text{MS}}$ bottom mass**

$$\hat{\Gamma}(h \to b\bar{b}) \overset{M_h \gg m_b}{=} \frac{3G_F}{4\sqrt{2}\pi} M_h \left(\bar{m}_b^{\overline{\text{MS}}}(M_h)\right)^2 \left[1 + 5.67\frac{\alpha_s(M_h)}{\pi} + \cdots\right] \qquad \textbf{(3)}$$



Figures from [Djouadi '05]

Theoretical uncertainties: sources and estimates

- Meaningful theory calculations relate physical observables (e.g. M_W, M_Z, α_{em}, G_F, …)

  Let's write in general: $O = \mathcal{F}(I_1, I_2, \cdots)$ *with I_1, I_2, … input parameters*

- 2 main sources of theoretical uncertainties:

  (1) Unknown higher-order and/or subleading contributions (in some step of the calculation)

  - *Example 1*: *O* is computed with a fixed-order calculation as

$$O = \underbrace{\mathcal{F}^{(0)}(I_1, I_2, \cdots)}_{\text{tree level}} + \underbrace{\frac{1}{16\pi^2}\mathcal{F}^{(1)}(I_1, I_2, \cdots)}_{\text{one loop}} + \underbrace{\frac{1}{(16\pi^2)^2}\mathcal{F}^{(2)}(I_1, I_2, \cdots)}_{\text{two loops}} + \cdots$$

  → perturbative expansion *truncated* at some order, higher-orders are unknown

  - *Example 2*: *O* computed with an Effective-Field-Theory calculation, in terms of parameter g_A obtained from EFT matching, i.e. $O = \tilde{\mathcal{F}}^{(0)}_{\text{EFT}}(I_1, g_A, \cdots) + \frac{1}{16\pi^2}\tilde{\mathcal{F}}^{(1)}_{\text{EFT}}(I_1, g_A, \cdots) + \cdots$

  with EFT matching of g_A: $g_A = \mathcal{G}^{(0)}(I_1, I_2, \cdots, \Lambda_{\text{match.}}) + \frac{1}{16\pi^2}\mathcal{G}^{(1)}(I_1, I_2, \cdots, \Lambda_{\text{match.}})$

  → missing higher-orders in EFT calc. + in matching + uncertainty from choice of matching scale Λ_{match}

How to estimate their effect? → renormalisation scheme conversions
→ variations of renormalisation scale or matching scale

Theoretical uncertainties: sources and estimates

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  (2) Finite precision with which input parameters are known

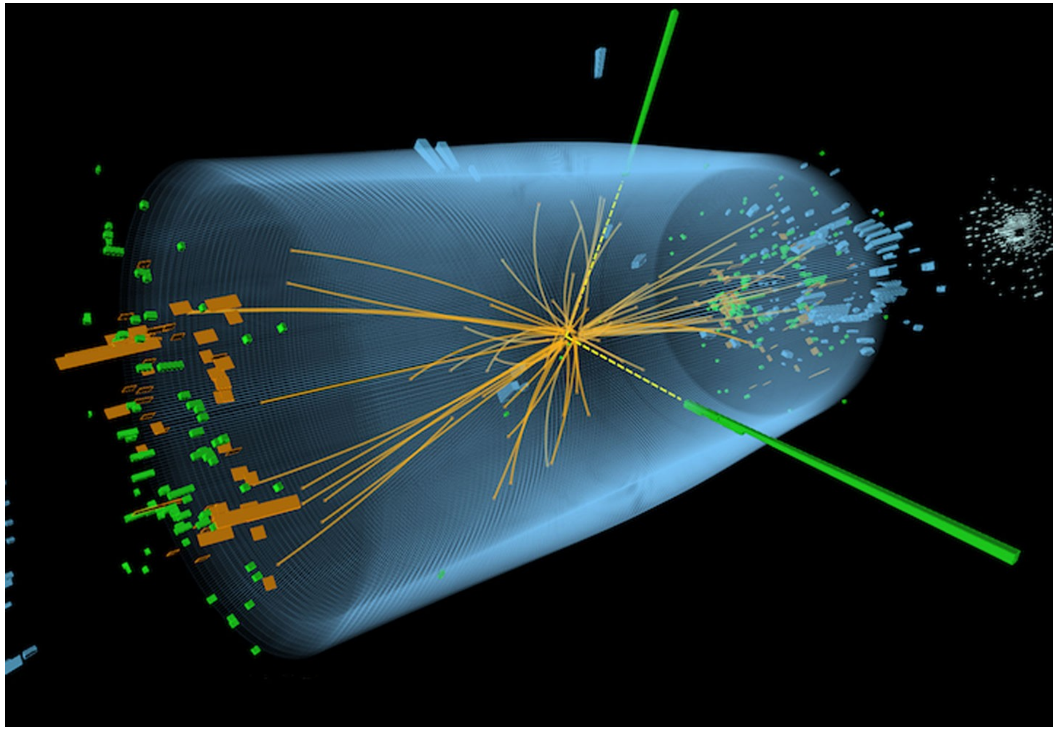
  → Assuming (for simplicity) that I_1, I_2, … are physical observables, their values are obtained from experimental measurements, that have an error/uncertainty

  How to estimate this effect? → error propagation (or simply repeat calculation with varied inputs)

e.g. $$\Delta O = \sqrt{\left(\frac{\partial \mathcal{F}}{\partial I_1}\right)^2 (\Delta I_1)^2 + \left(\frac{\partial \mathcal{F}}{\partial I_2}\right)^2 (\Delta I_2)^2 + \cdots}$$

Experimental errors on I_1, I_2, …

Higgs measurements and precision calculations

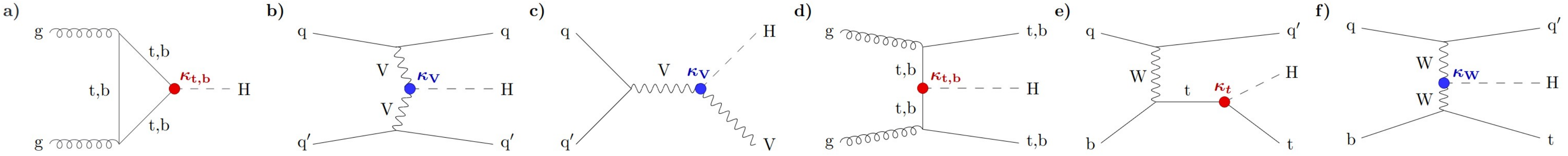


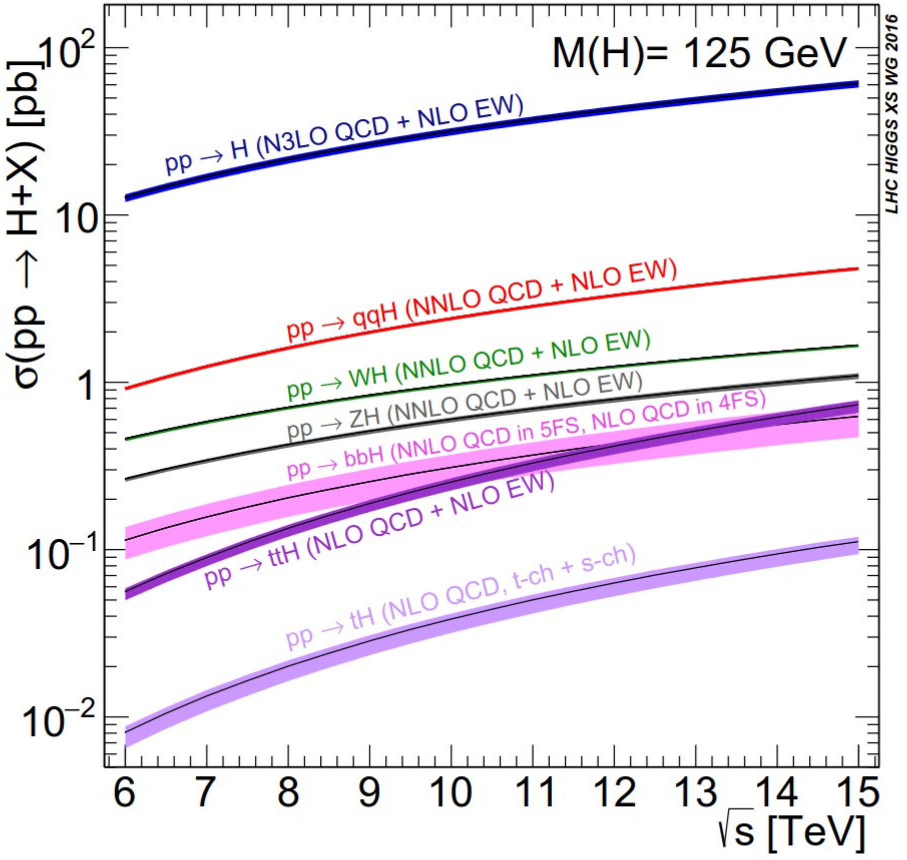
A $h \rightarrow \gamma\gamma$ event at CMS

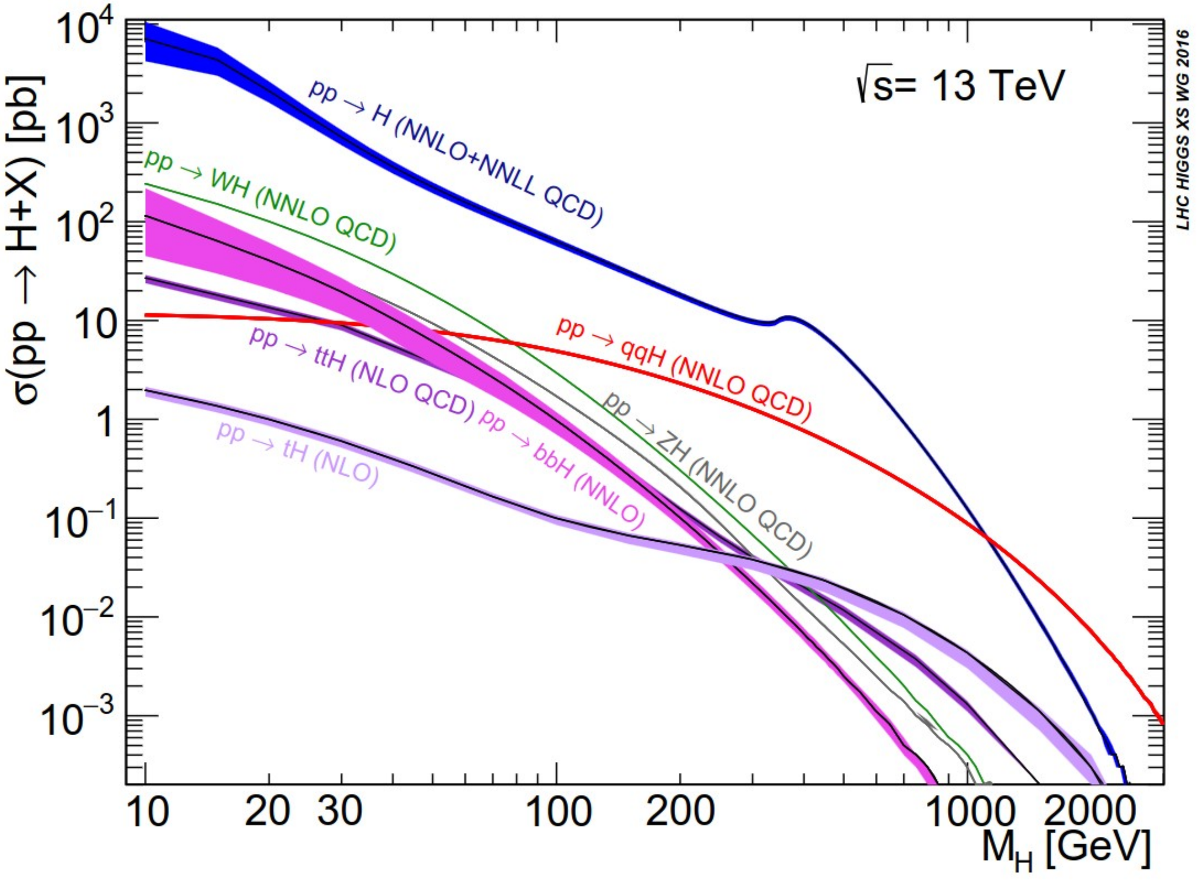
Higgs production at LHC

Diagrams from [CMS Nature '22],
Plots from [LHC Higgs WG '16]
See also reviews of [Djouadi '05]

- (Single-)**Higgs production channels at LHC**



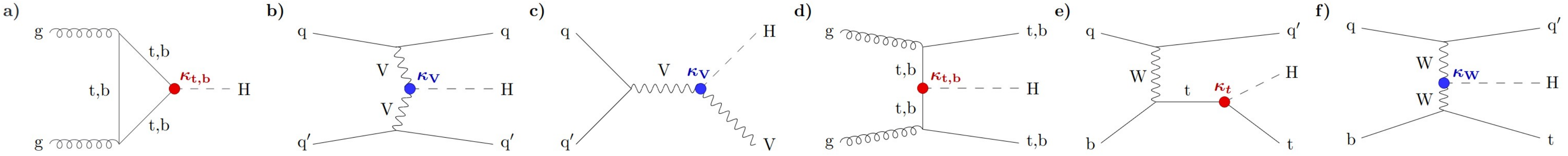


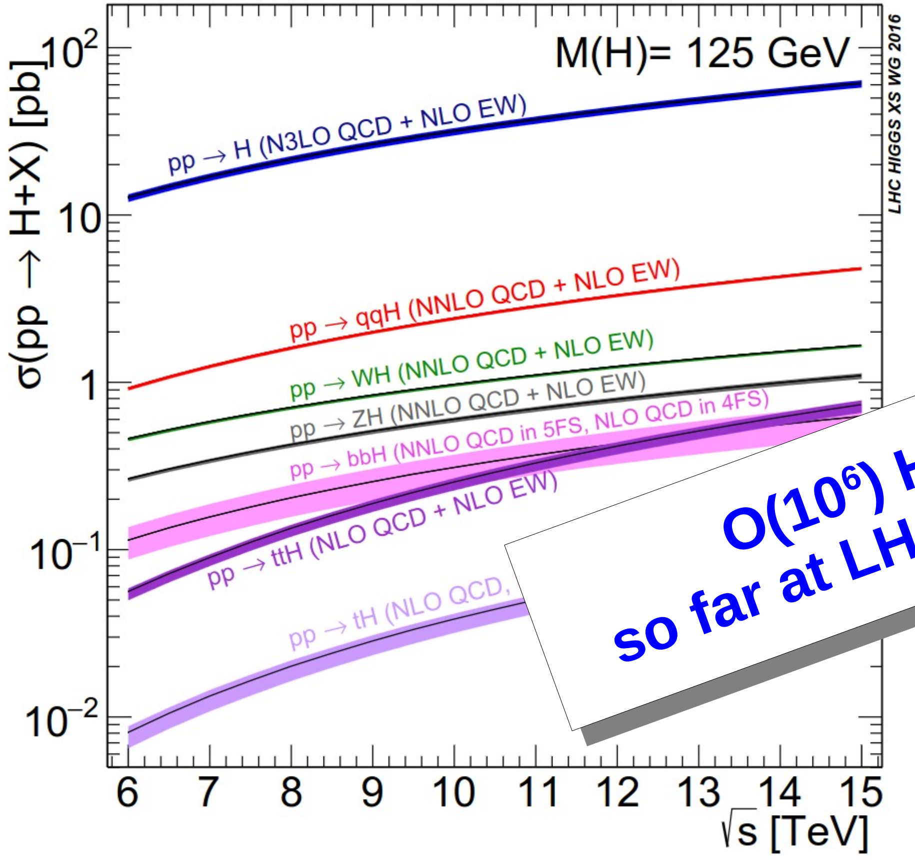


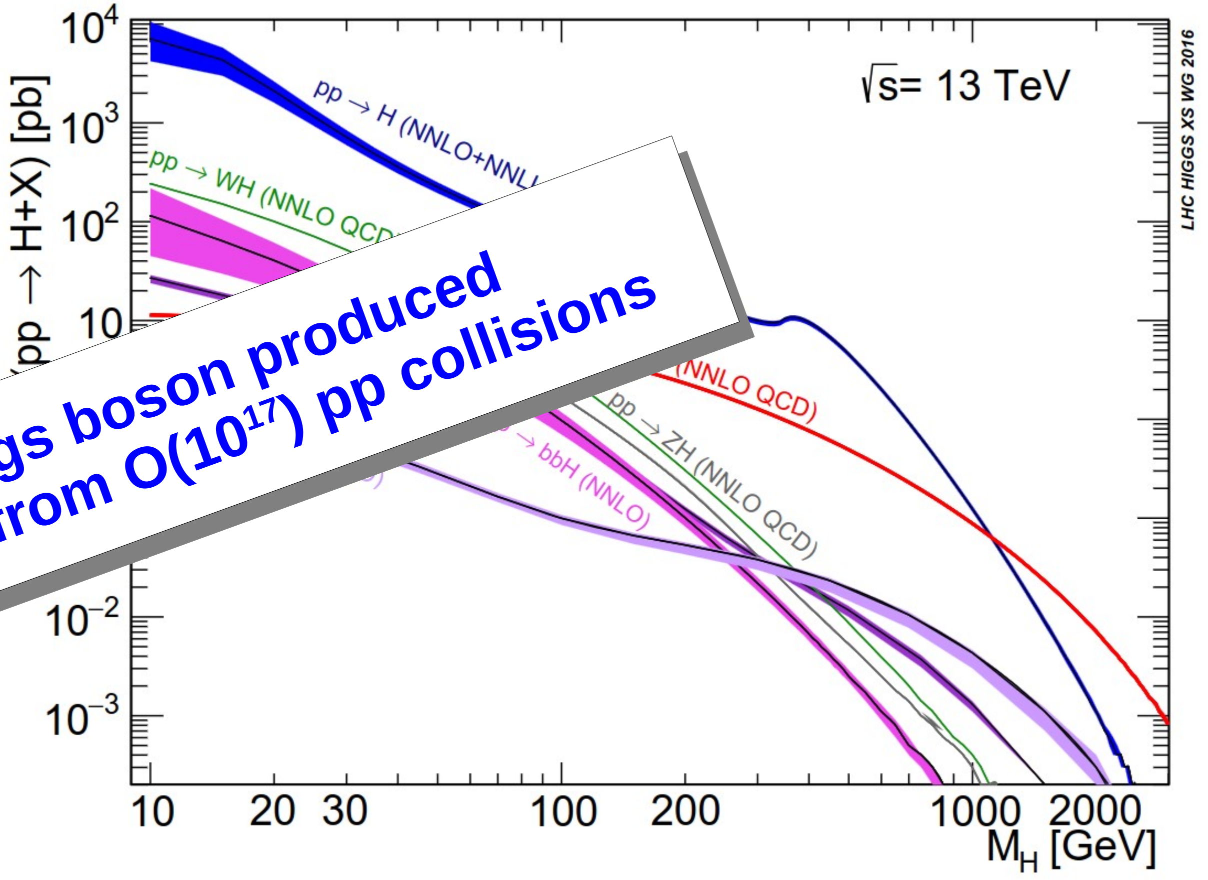
Higgs production at LHC

Diagrams from [CMS Nature ‘22],
Plots from [LHC Higgs WG ‘16]
See also reviews of [Djouadi ‘05]

- (Single-)**Higgs production channels at LHC**







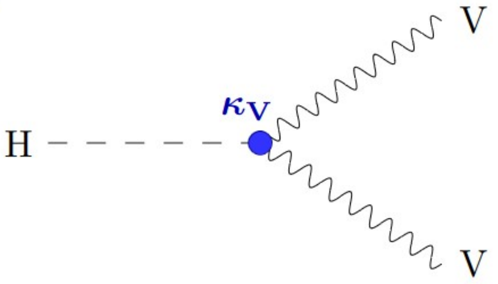
O(10^6) Higgs boson produced so far at LHC, from O(10^{17}) pp collisions

Higgs decay channels

- **Decay channels**

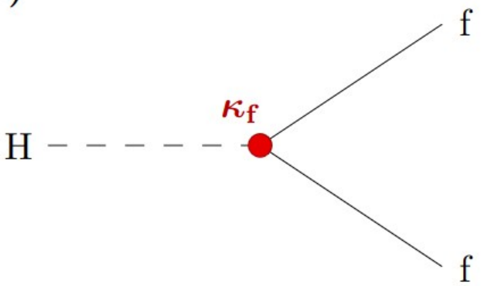
Tree-level decays:

g)



(V=W, Z)

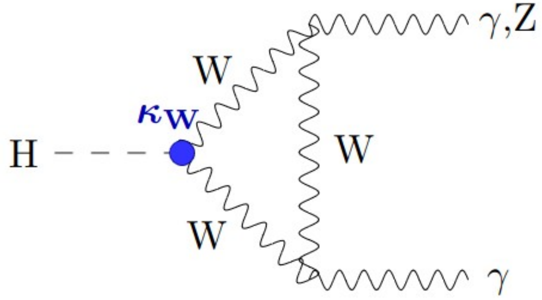
h)



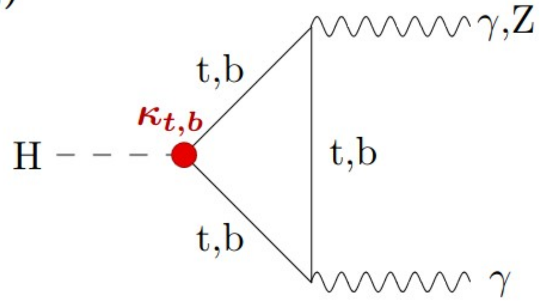
(f=b, τ, μ, ...)

Loop-induced decays:

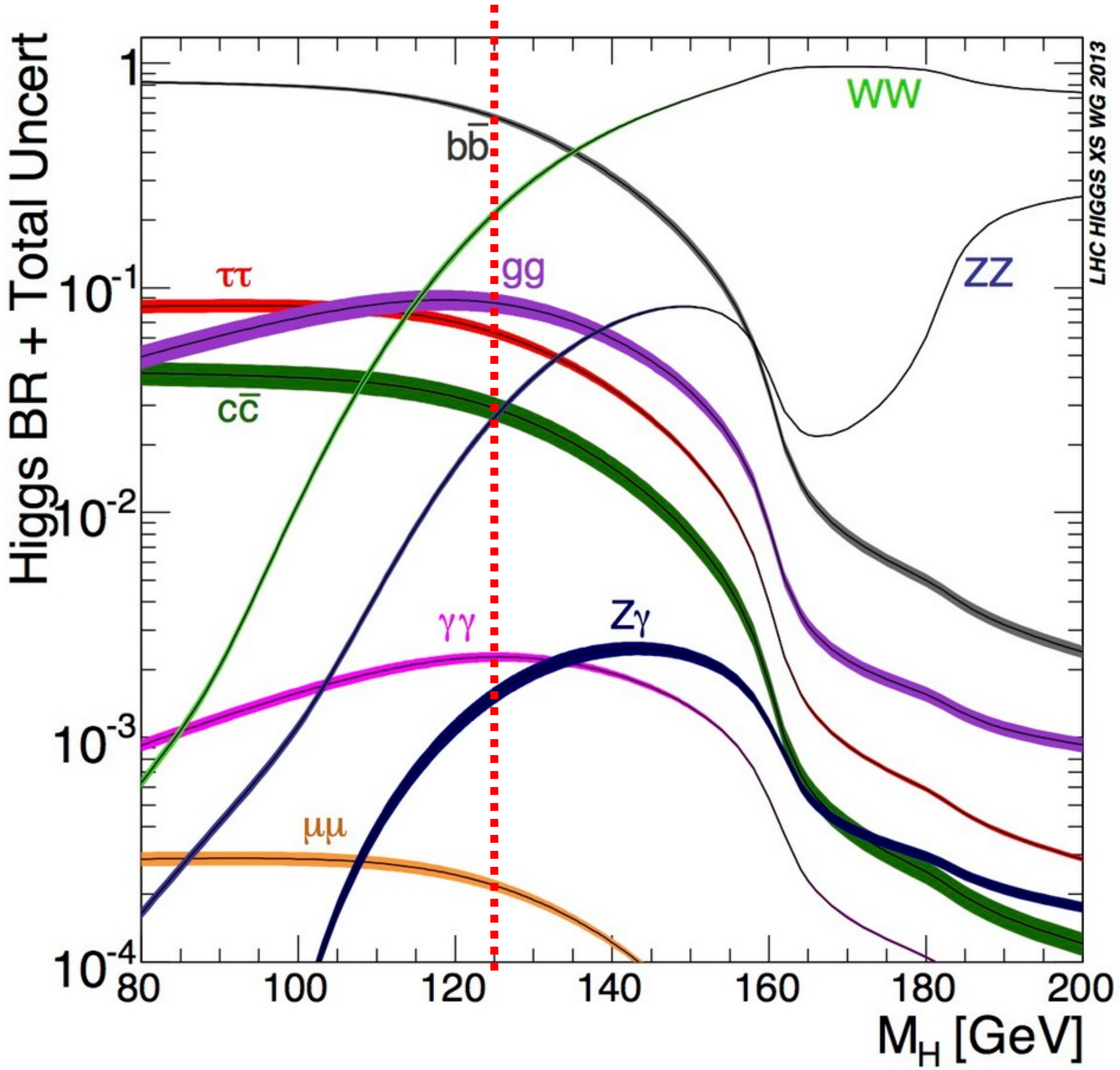
i)



j)

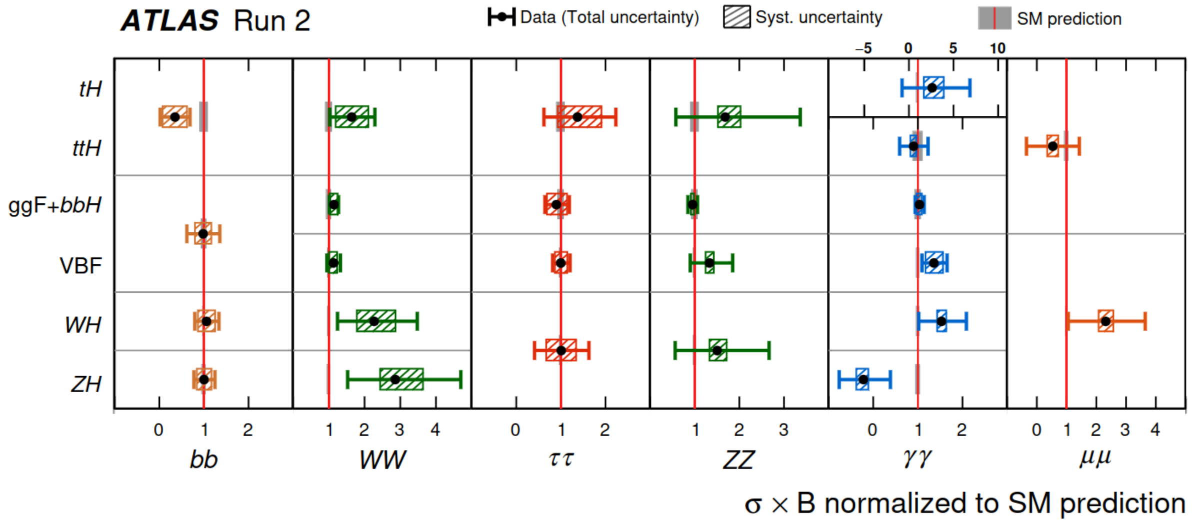


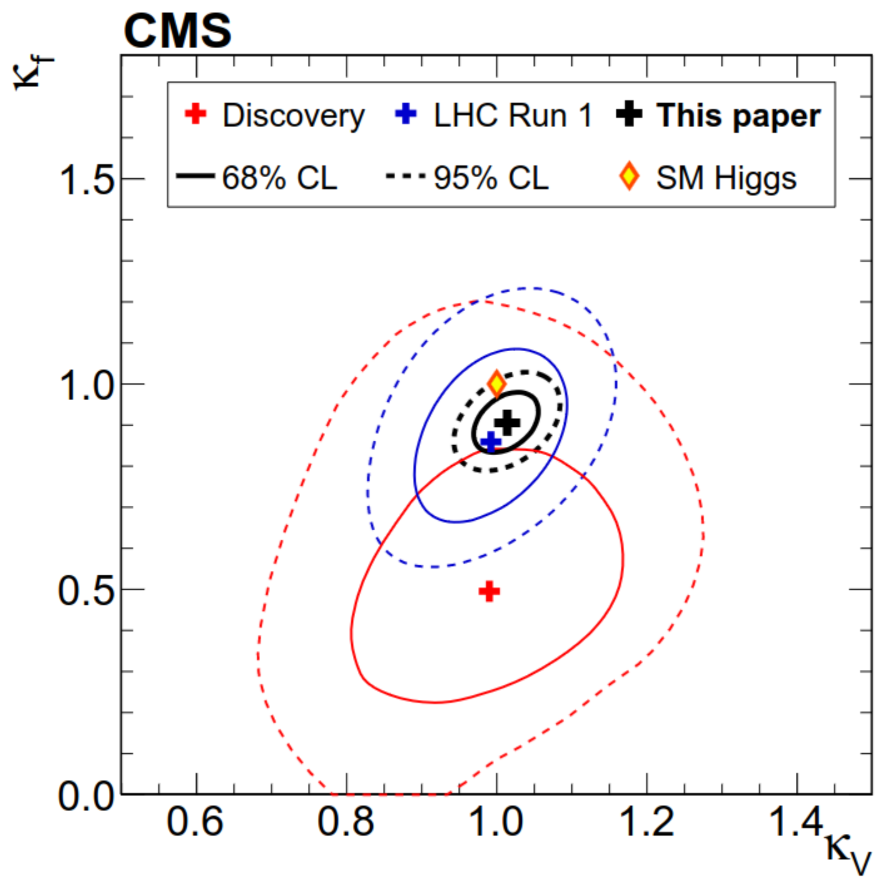
Diagrams from
[CMS Nature '22]

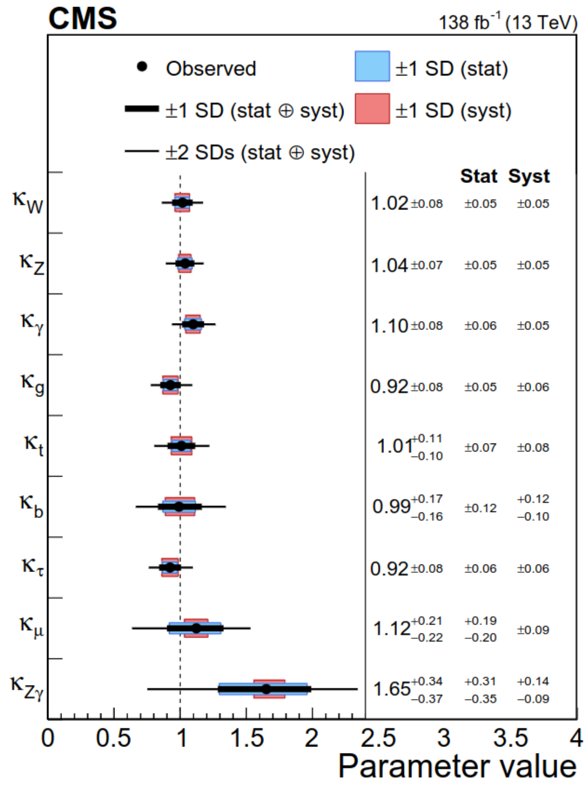


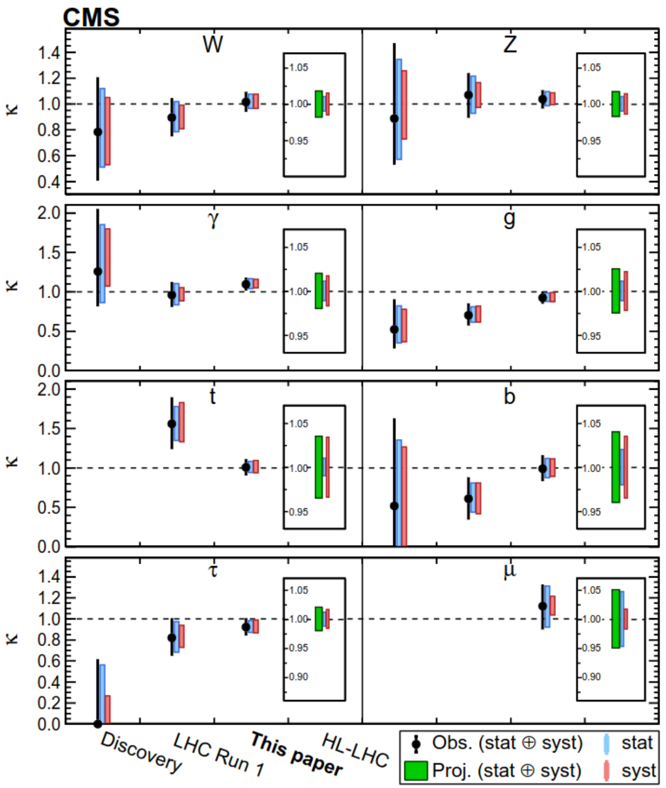
Example results of Higgs measurements

- Example results from ATLAS and CMS 2022 ``10-year of the Higgs discovery” Nature papers, for **coupling modifiers** (*bottom*) or **signal strengths** (*right*)





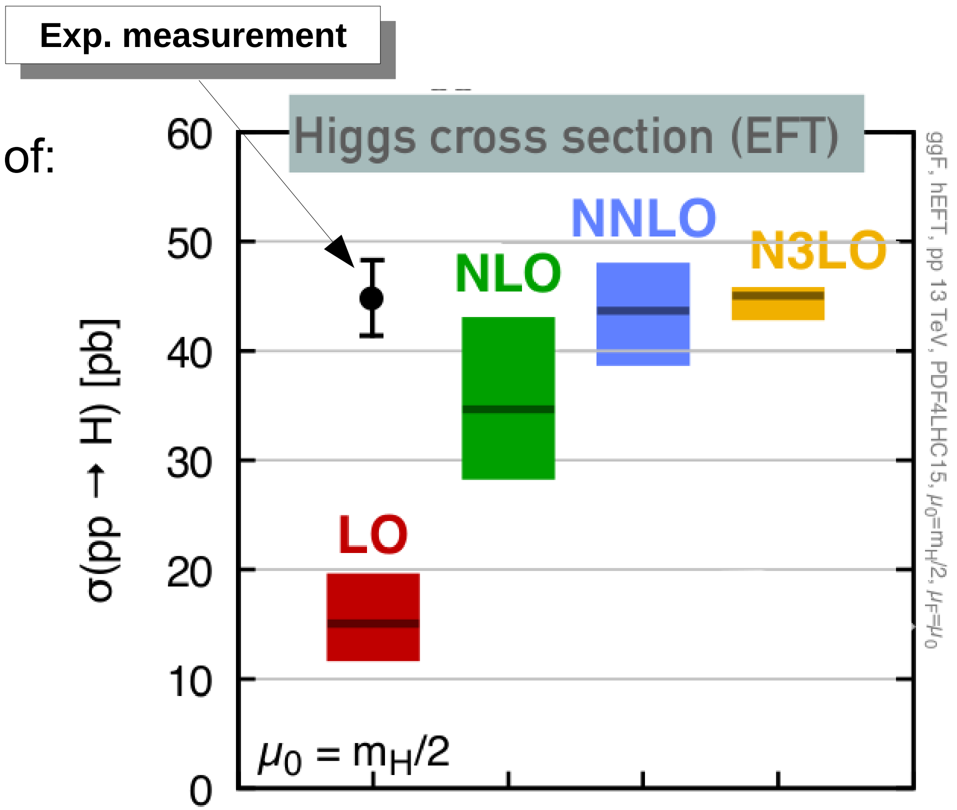




Interpreting experimental results

- Comparison between experiment and theory carried out at the level of:
  - Signal strengths
  - κ parameters (signal strength modifiers)
  - Simplified Template Cross-Sections (STXS)
  - Fiducial cross sections
  - Coefficients of EFT operators

- Requires **high-precision theoretical predictions** (with level of accuracy at least matching that of experimental results)
  → both in SM and BSM theories
  → huge efforts from precision calculation communities
  (QCD, EW, BSM)

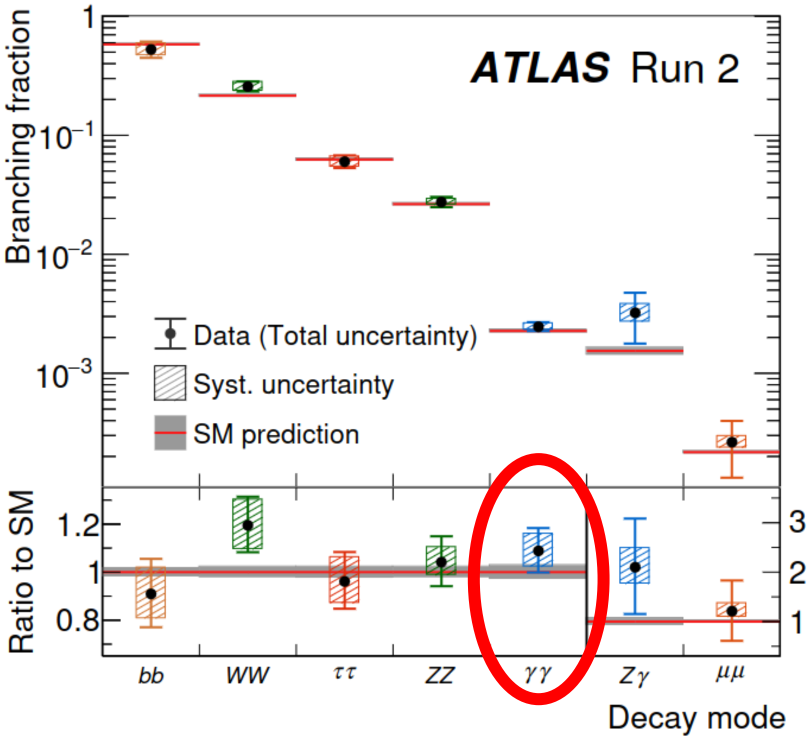


Total cross section for (inclusive) single-Higgs production, in heavy top limit ($m_t \to +\infty$)

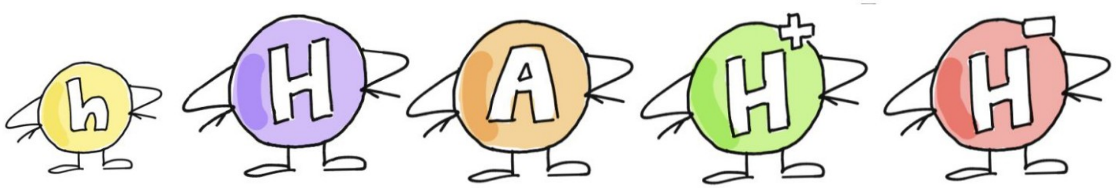
Figure taken from [Weiglein ‘22], itself from [Wiesemann ‘22], based on results from [Anastasiou et al. '15], [Mistlberger '18]

- **Public tools to confront model predictions with experimental results**:
  - HiggsSignals (signal strengths, STXS) [Bechtle et al ‘13, ‘20] → now included in HiggsTools [Bahl et al ‘22]
  - Lilith (signal strengths) [Bernon, Dumont ‘15], [Kraml et al ‘19], [Bertrand et al ‘20]

An example calculation of Higgs properties in a BSM model: leading two-loop corrections to Γ(h→ γγ) in the Inert Doublet Model



The Inert Doublet Model



Drawing by [K. Radchenko Serdula '24]

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG) \end{pmatrix} \qquad \text{and} \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}$$

- **Unbroken Z_2 symmetry** $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$

$$\begin{aligned} V^{(0)}_{\text{IDM}} = \mu_1^2|\Phi_1|^2 + \mu_2^2|\Phi_2|^2 \\ + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\left((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\right) \end{aligned}$$

- Inert scalars H, A, $H^\pm$: charged under Z_2 symmetry (Z_2-odd)

- **Model parameters**:
  3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale μ_2, inert doublet quartic self-coupling λ_2

- **Lightest inert scalar = Dark Matter candidate**
  → *assume H here*

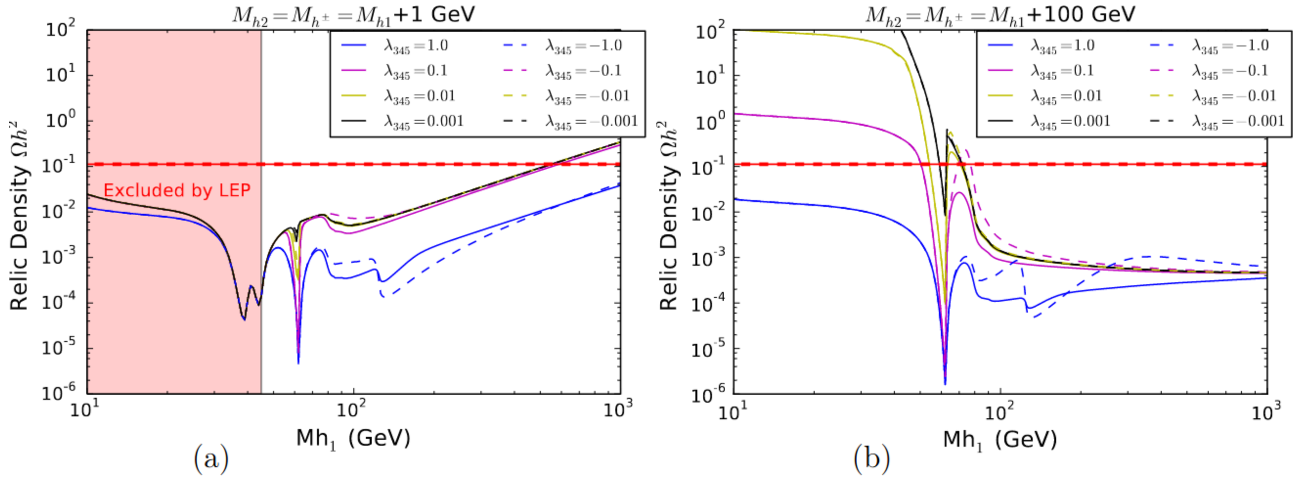
Dark Matter in the Inert Doublet Model

- DM (*H*) relic density obtained via freeze-out mechanism, while evading current detection bounds

- 2 possible scenarios:
  → **"Higgs resonance scenario"**
  $m_H \sim m_h/2$
  → **"Heavy Higgs scenario"**
  $m_H \gtrsim 500$ GeV

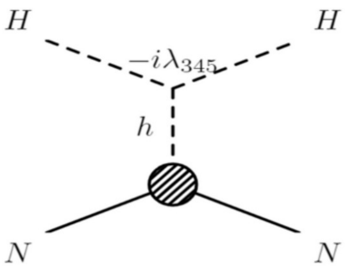
- IDM testable at current and future experiments via
  - DM direct and indirect searches
  - direct searches at colliders

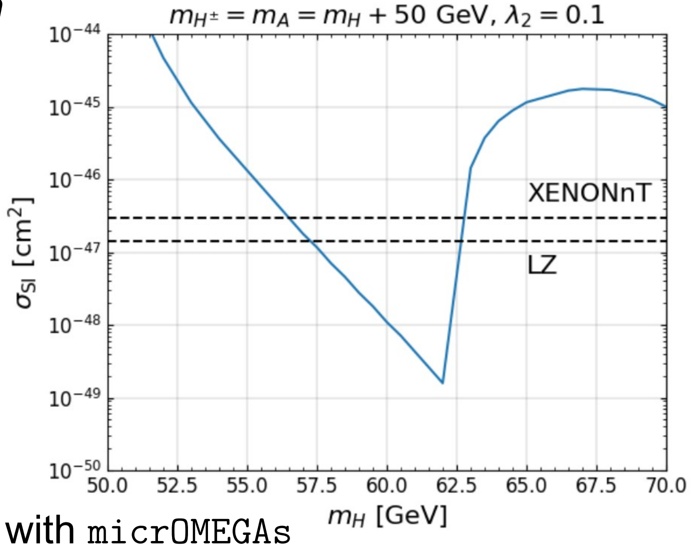
  (see also [JB, Gabelmann, Robens, Stylianou '24])
  - *precision/indirect tests*
    → properties of h_{125}



[Belyaev et al. '16]

Direct detection bounds around Higgs resonance region





Plot made with `micrOMEGAs`
[Bélanger et al. '18]

Dark Matter in the Inert Doublet Model

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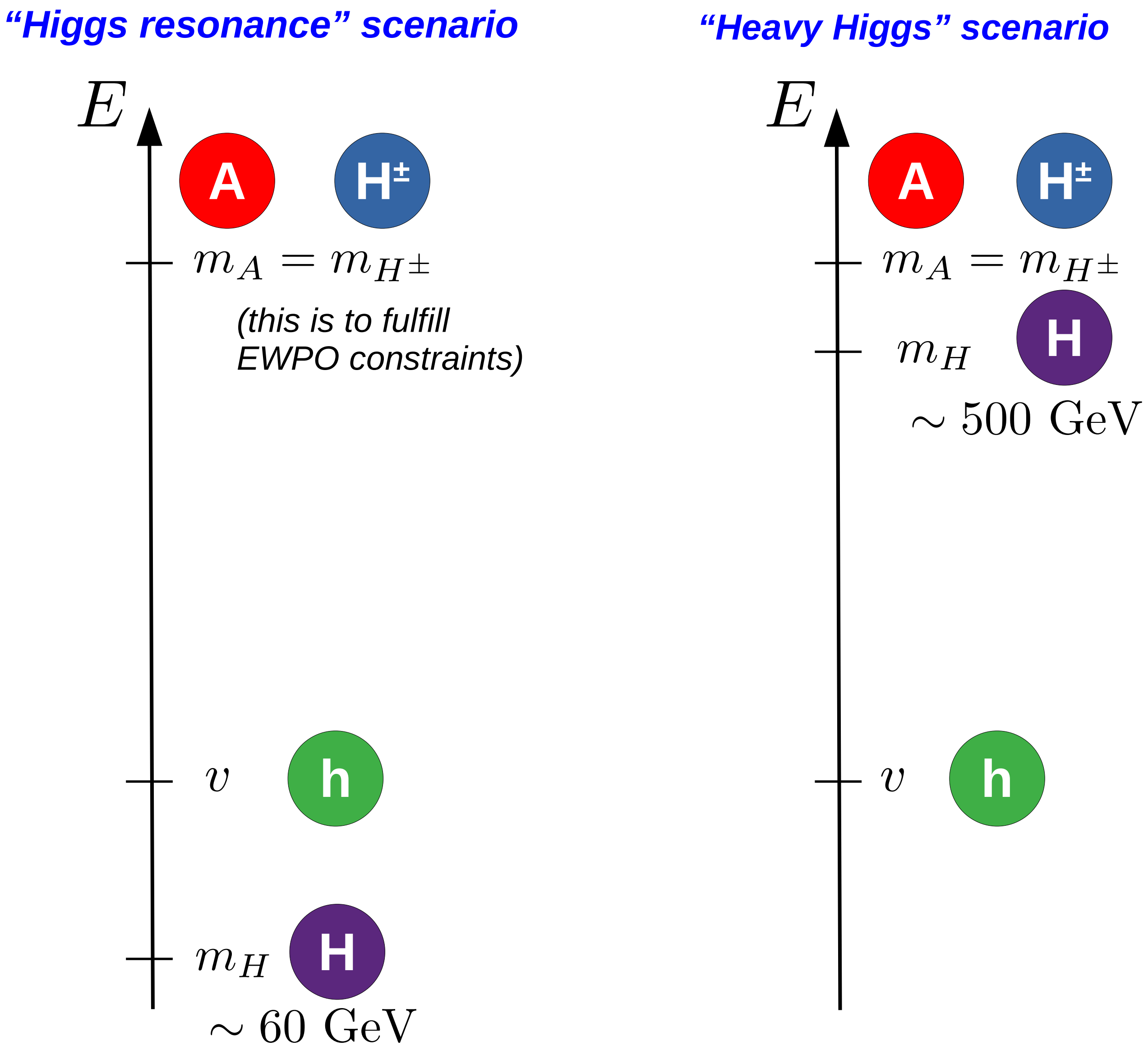
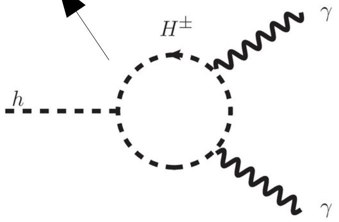


Higgs decay to two photons: existing one-loop results

- DM scenarios of IDM investigated via Higgs properties at one loop (1L) in [Kanemura, Kikuchi, Sakurai '16]
- Additional charged inert Higgs → Higgs decay to 2 photons especially important!

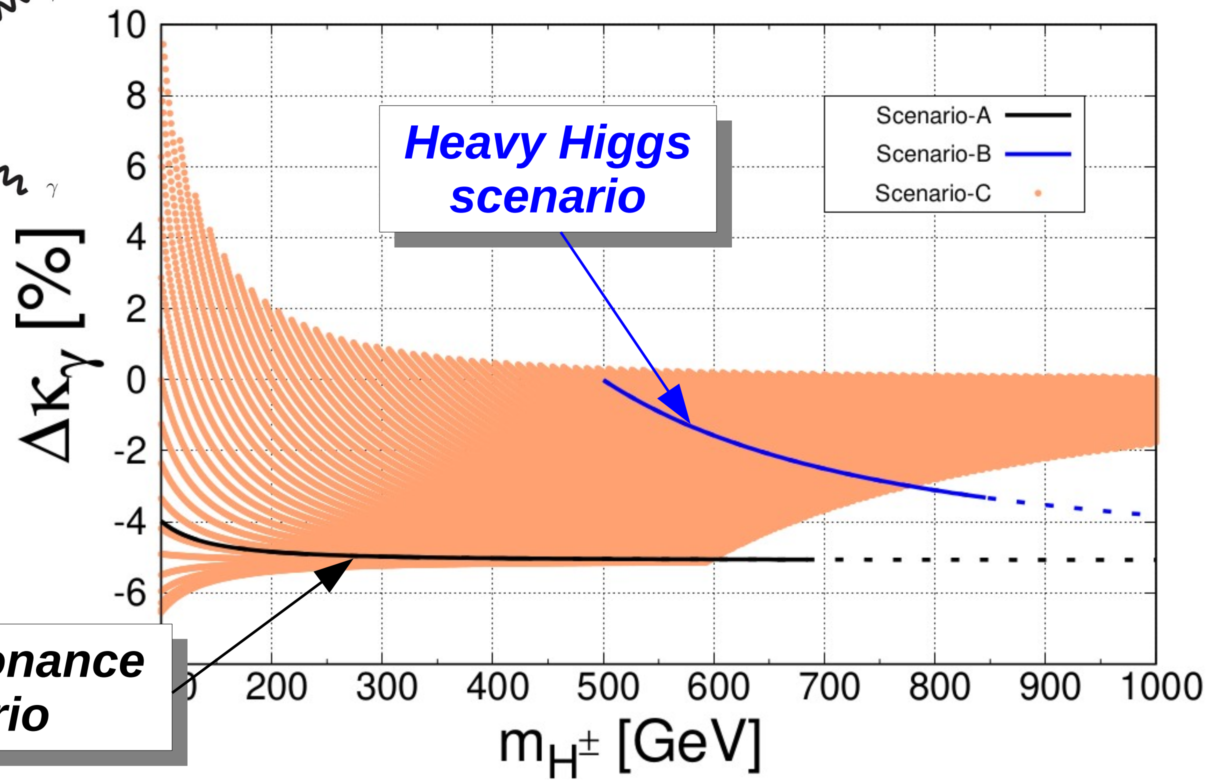
$$\Gamma[h \to \gamma\gamma] \underset{m_h \ll m_{H^\pm}}{\simeq} \frac{\sqrt{2} G_F \alpha_{\mathrm{EM}}^2 m_h^3}{64\pi^3} \left| -\frac{1}{6}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) + \sum_f Q_f^2 N_c^f I_f[m_h^2] + I_W[m_h^2] \right|^2 \qquad \text{with } m_{H^\pm}^2 = \mu_2^2 + \lambda_3 v^2 \,,$$

I_f, I_W: fermion/W-boson loops (SM-like)



- **Charged Higgs contribution:**
  Compensation between mass dependence of coupling ($\lambda_3=2(m_{H\pm}^2-\mu_2^2)/v^2$) and of loop function ($C_0 \sim 1/m_{H\pm}^2$)
  → does not decouple when increasing $m_{H\pm}$!

- h→γγ is a loop-induced decay, i.e. ***1L is only leading order (LO)***
  → **What about 2L (NLO) corrections?**



Higgs Low-Energy Theorem

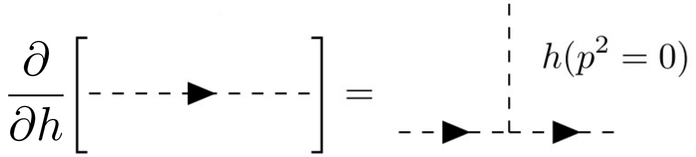
- Calculation of 2L 3-point functions with external momenta not possible in general

- Assuming m_h << heavy BSM scalar masses, we can employ a **Higgs Low-Energy Theorem** (see e.g. [Kniehl, Spira '95])
- Compute **effective Higgs-photon coupling** $C_{h\gamma\gamma}$ of the form

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} C_{h\gamma\gamma} h F^{\mu\nu} F_{\mu\nu}$$

  by taking derivative of (unrenormalised) photon self-energy w.r.t Higgs field

$$C_{h\gamma\gamma} = \frac{\partial}{\partial h} \Pi_{\gamma\gamma}(p^2 = 0)\Big|_{h=0} \qquad \text{where } \Sigma^{\mu\nu}_{\gamma\gamma}(p^2) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_{\gamma\gamma}(p^2)$$

- Schematically: 

- Neglects incoming momentum on Higgs leg, but valid for m_h << $m_{H,A,H\pm}$

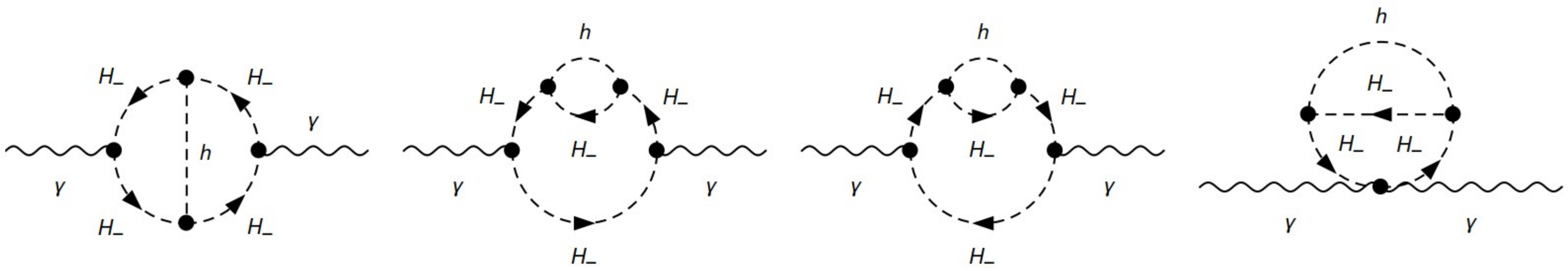
Computing two-loop BSM corrections to Γ(h→ γγ)

- **All known SM contributions**:
  - QCD up to 3L [Djouadi '08] (+ refs. therein)
  - EW SM-like to full 2L [Degrassi, Maltoni '05], [Actis et al. '09]

- Our new calculation: **leading two-loop BSM contributions**
  - genuine, dominant, 2L contributions involving inert scalars (+ SM-like scalars and/or gauge bosons)
  - purely scalar and fermion-scalar contributions to (1L)^2 terms from external-leg and VEV renormalisation

$$C_{h\gamma\gamma}^{(2),\,\mathrm{IDM}} = C_{h\gamma\gamma}^{\mathcal{O}(\lambda_3^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4+\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4-\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}(\lambda_2)} + C_{h\gamma\gamma}^{\mathrm{ext.\text{-}leg.+VEV}}$$

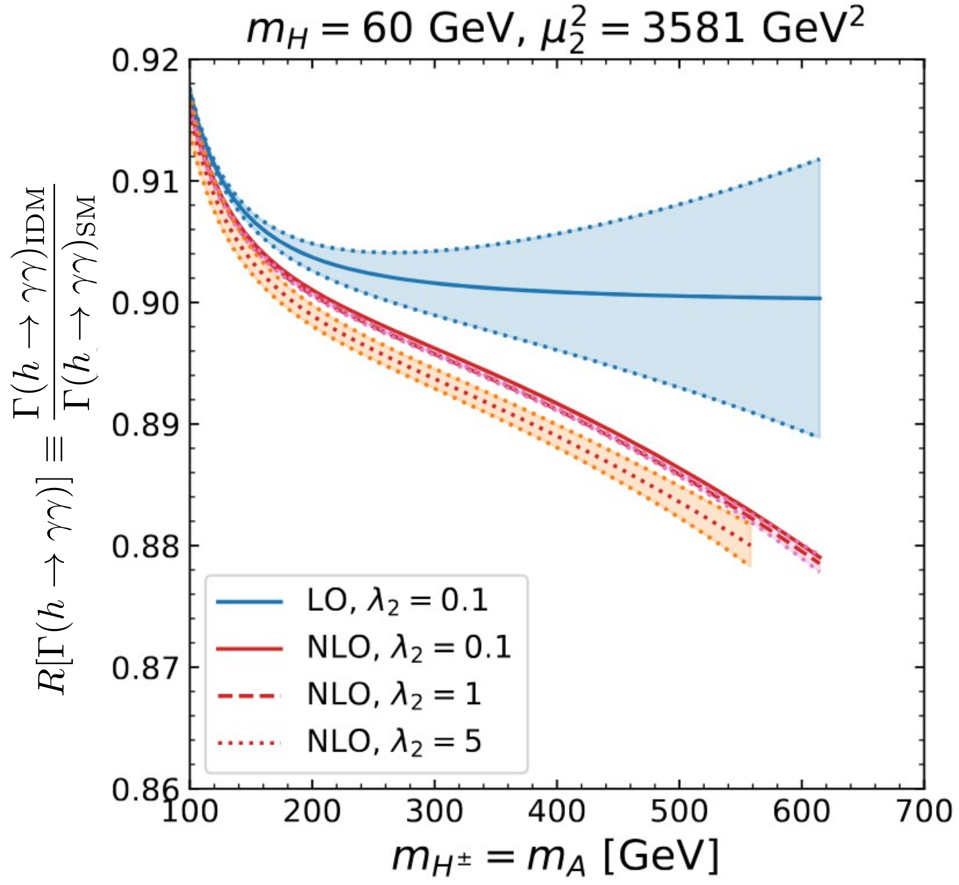
Example:
O(λ_3^2) diagrams

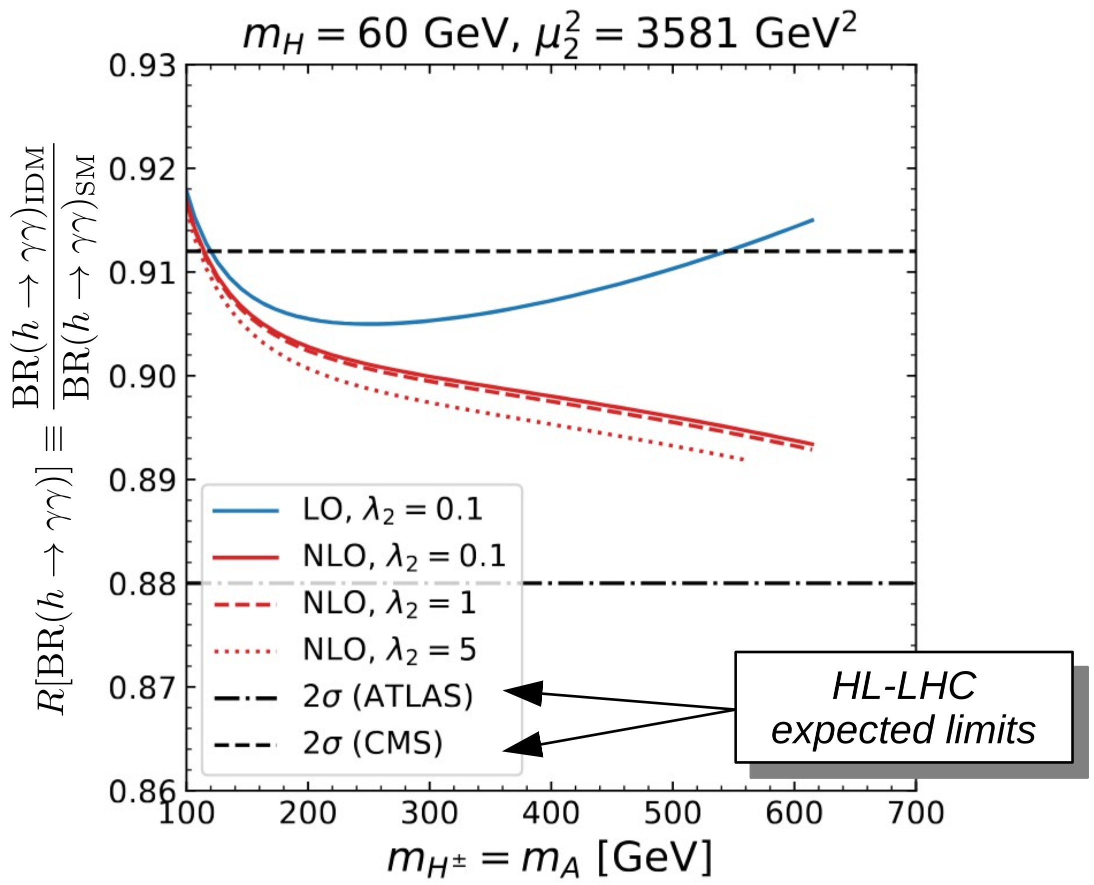


- Photon self-energy diagrams generated with `FeynArts`, computed with `FeynCalc` and `Tarcer`, reduced to (limits of) integrals known analytically; then derivative w.r.t. h taken

Results for the Higgs resonance scenario

[Aiko, JB, Kanemura '23]





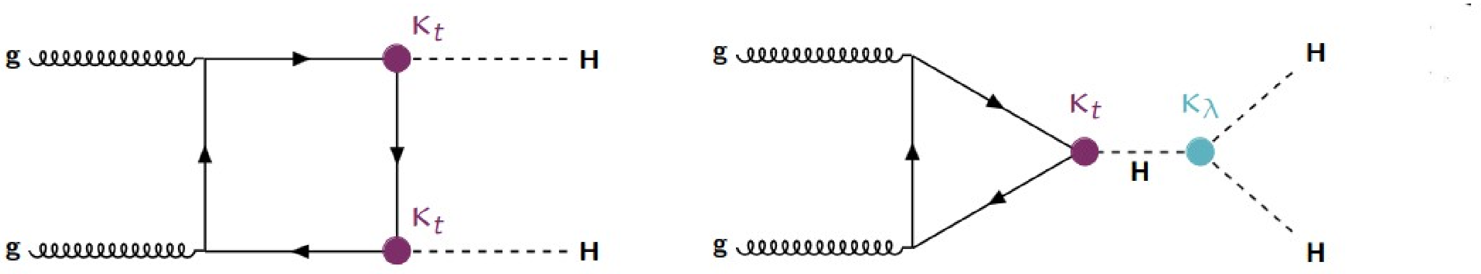
Inclusion of two-loop (NLO) corrections significantly reduces the theoretical uncertainty

- **Almost entire scenario** (for $m_{H\pm} > 120$ GeV) **can be ruled out if no deviation is found in h→γγ!**
- **Proper interpretation of experimental results requires inclusion of two-loop corrections!**

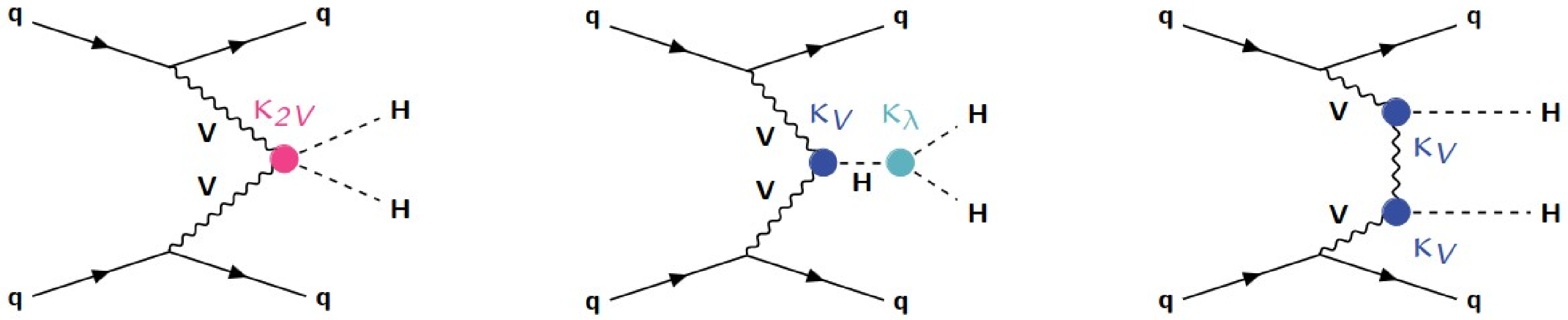
Di-Higgs production:
Theory predictions and uncertainties

Different channels for di-Higgs production

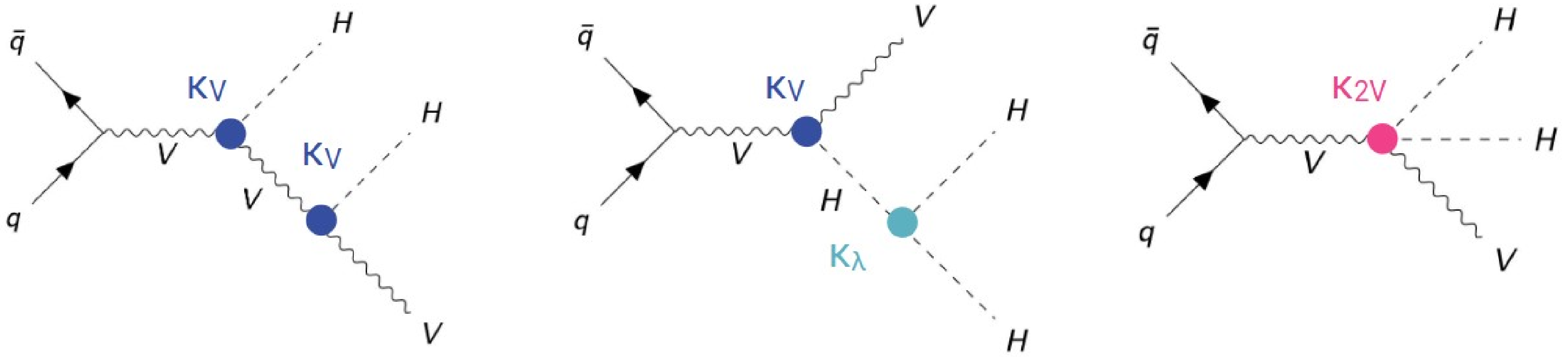
gluon-fusion
$\sigma_{ggF}(pp \to HH) = 31.05$ fb



VBF
$\sigma_{VBF}(pp \to HH) = 1.726$ fb

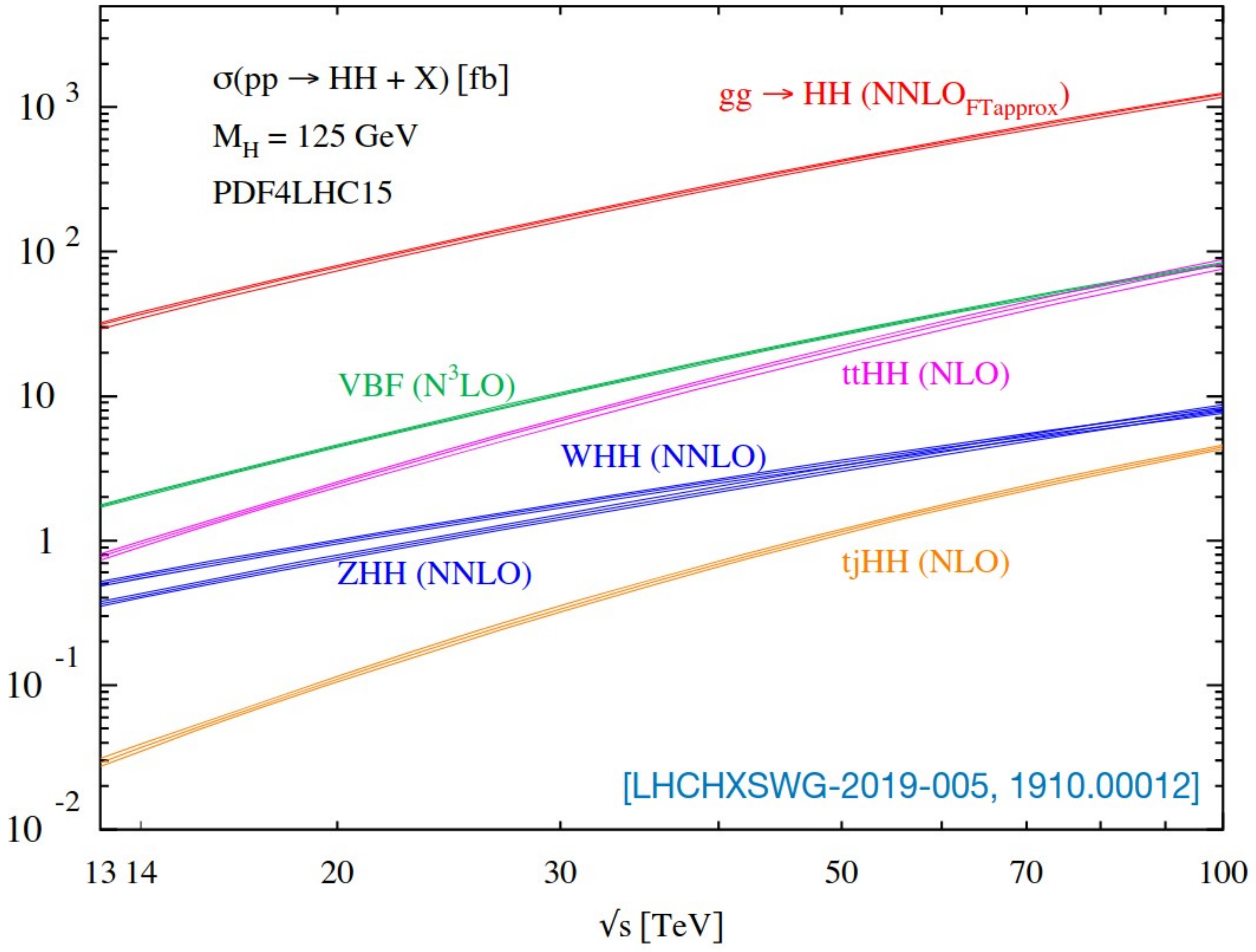


VHH
$\sigma_{VHH}(pp \to HH) = 0.86$ fb

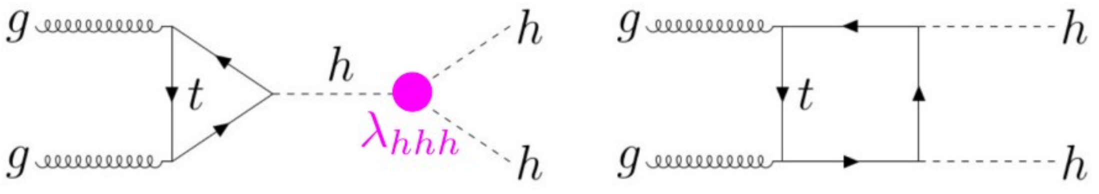


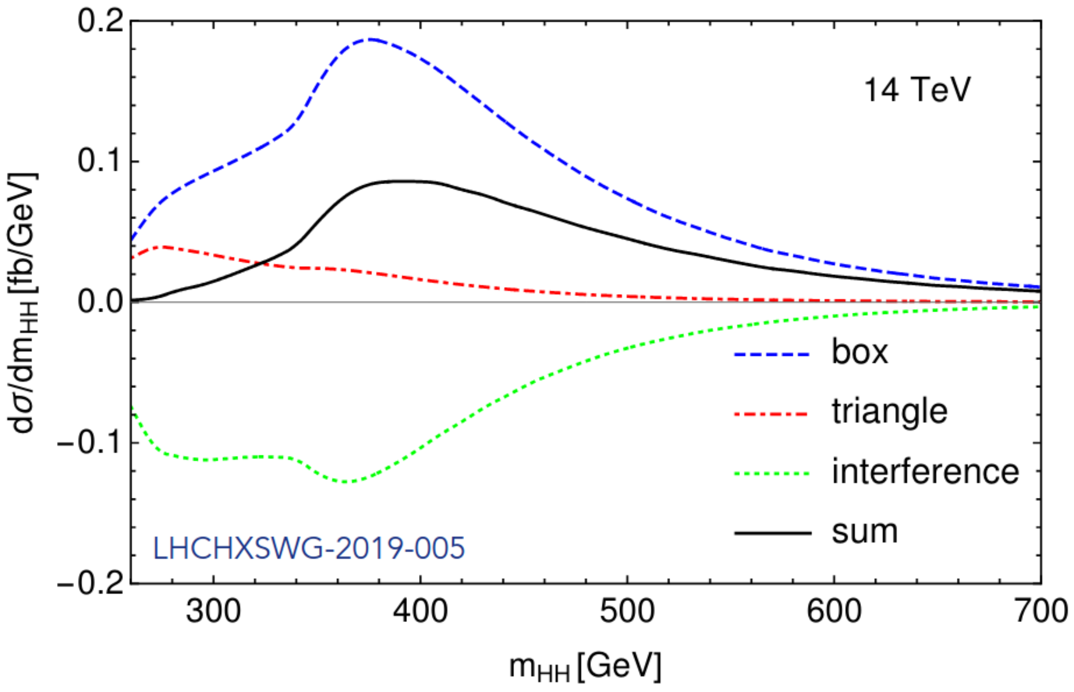
Slide by K. Leney @ HiggsDays 23

Different channels for di-Higgs production



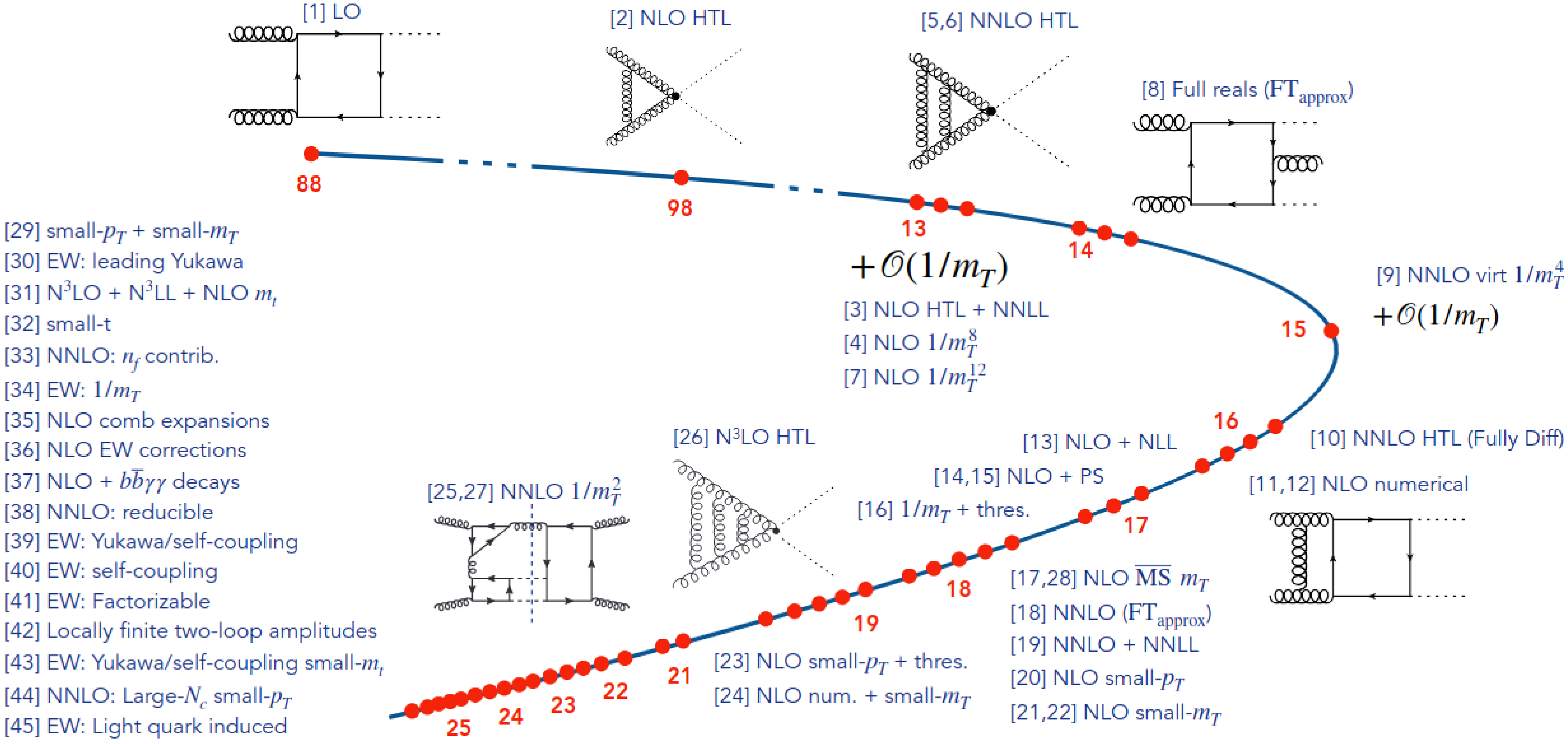
Leading channel: gluon fusion





Di-Higgs production via gluon fusion: theory progress in SM

Slide by S. Jones
(design by G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, Jones, Kerner, Luisoni, Vryonidou 17; [15] Jones, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Ajjath, Shao 22; [32] Davies, Mishima, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser 23; [34] Davies, Schönwald, Steinhauser, Zhang 23; [35] Bagnaschi, Degrassi, Gröber 23; [36] Bi, Huang, Huang, Ma Yu 23 [37] Li, Si, Wang, Zhang, Zhao 24; [38] Davies, Schönwald, Steinhauser, Vitti 24; [39] Heinrich, Jones, Kerner, Stone, Vestner [40] Li, Si, Wang, Zhang, Zhao 24; [41] Davies, Schönwald, Steinhauser, Zhang 24; [42] C. Anastasiou, J. Karlen, G. Sterman, A. Venkata 24; [43] Davies, Schönwald, Steinhauser, Zhang 25; [44] Davies, Schönwald, Steinhauser 25; [45] Bonetti, Rendler, Bobadilla 25;

Di-Higgs production via gluon fusion: uncertainty budget

[Slide elements by S. Jaskiewicz]

Combination of NLO and N^{m}LO HTL yields:

- Scale uncertainty of: $+2.1\,\%$ / $-4.9\,\%$
- PDF $+\alpha_s$: $\pm 2.2\,\%$
- m_T approx: $\pm 2.7\,\%$

HWG HH Twiki

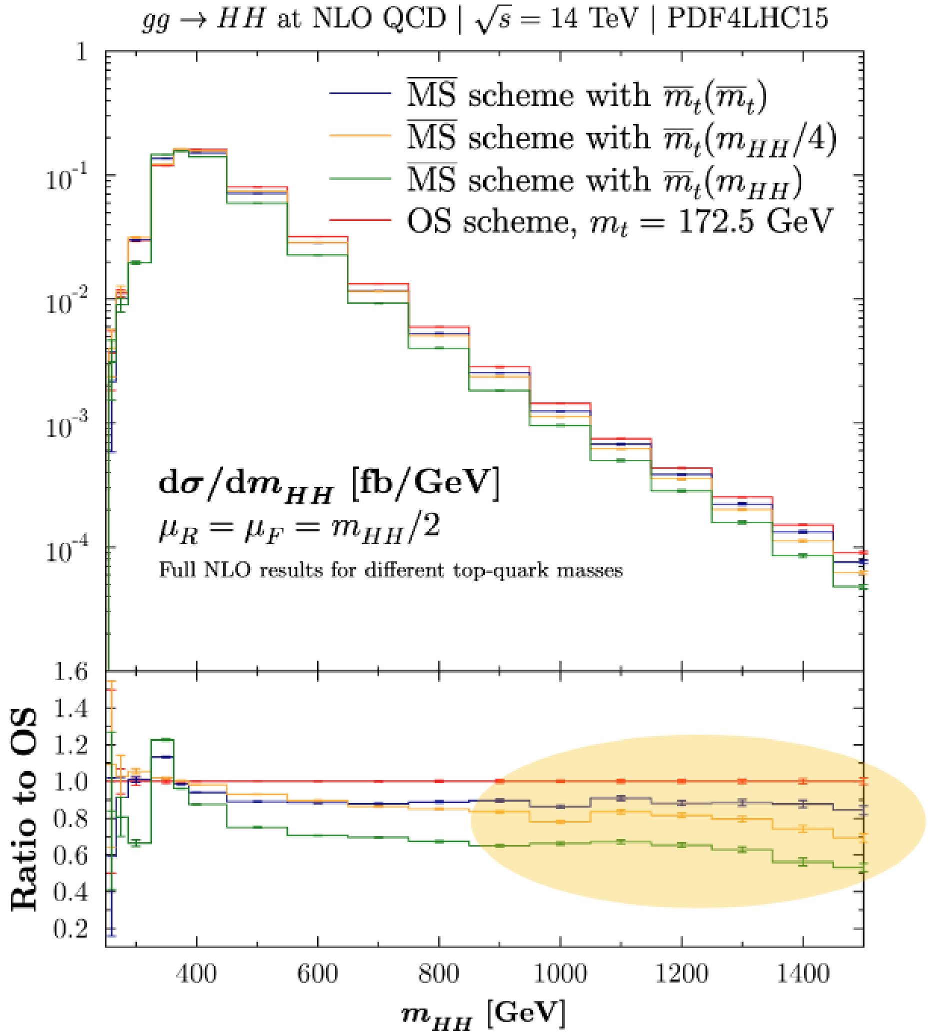
[Chen, Li, Shao, Wang 19, 19; Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli 18; de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; Maltoni, Vryonidou, Zaro 14; Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke 16; Dawson, Dittmaier, Spira 98; Glover, van der Bij 88]

Converting the top quark mass to the $\overline{\mathrm{MS}}$ scheme

$$m_t \to \overline{m}_t(\mu)\left(1 + \frac{\alpha_s(\mu)}{4\pi}C_F\left\{4 + 3\log\left[\frac{\mu^2}{\overline{m}_t(\mu)^2}\right]\right\}\right)$$

This leads to an additional uncertainty related to the choice of the top-quark mass scheme

$$\sqrt{s} = 13\text{ TeV}: \quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%}\text{ fb},$$
$$\sqrt{s} = 14\text{ TeV}: \quad \sigma_{tot} = 32.81(7)^{+4\%}_{-18\%}\text{ fb},$$

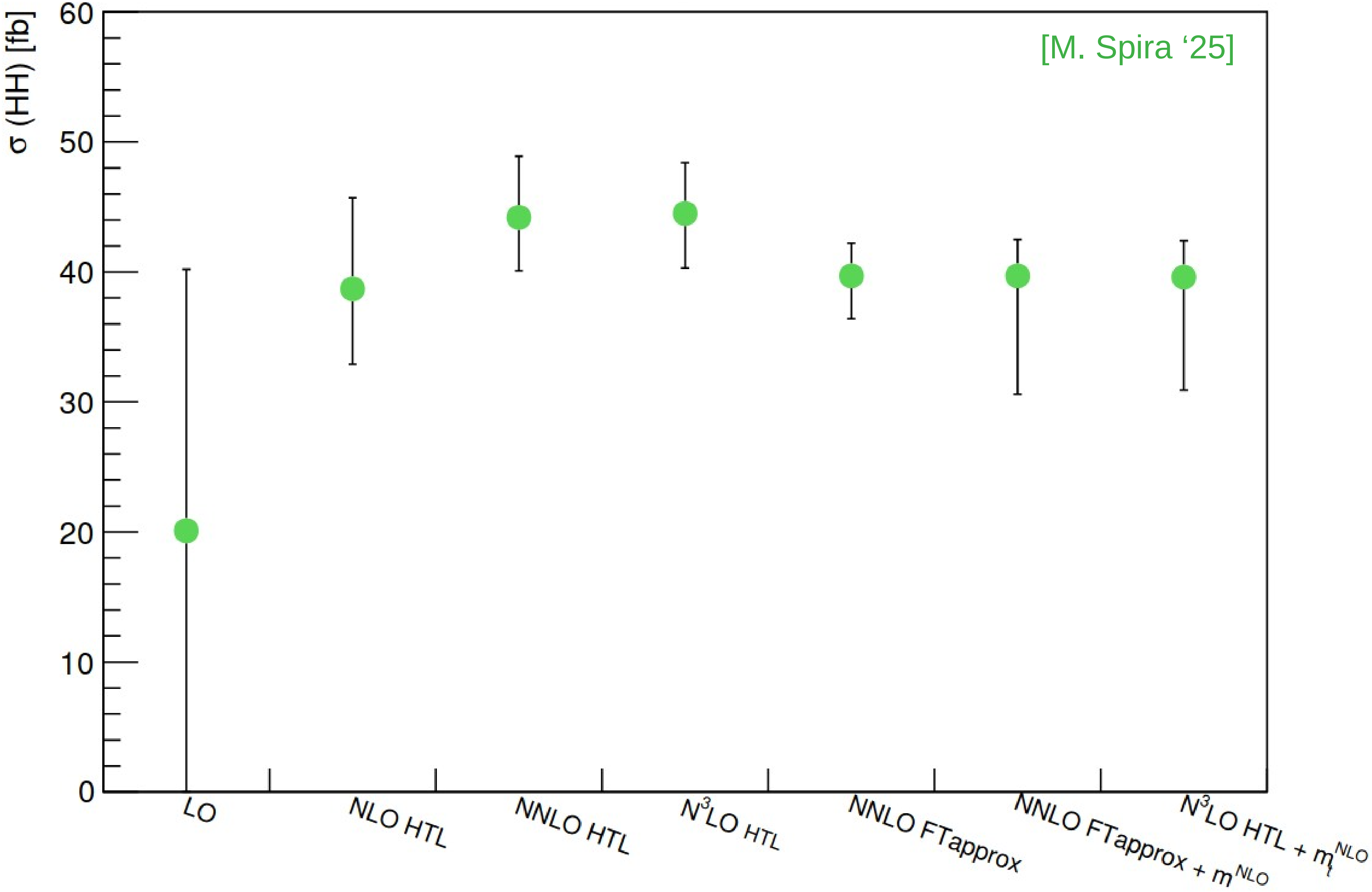


Large uncertainties in the high-energy limit

[J. Baglio, F. Campanario, S. Glaus, M. Muehlleitner, J. Ronca, M. Spira, J. Streicher, 2003.03227, 2008.11626]

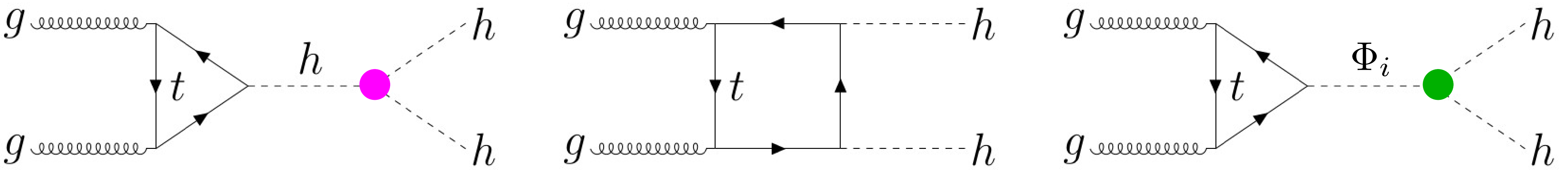
- *Currently*: attempts at resumming these large logs by EFT for high-energy limit (SCET = soft collinear effective theory) → [Kaskiewicz et al. '25] → improvement but new uncertainties from matching scale

Di-Higgs production via gluon fusion: evolution of uncertainty



Di-Higgs production in BSM models

➢ Leading order (LO) diagrams (involving top quark) in BSM models

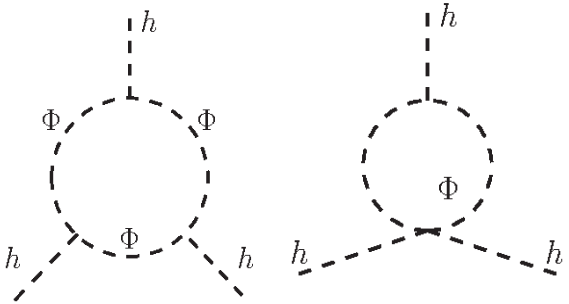


"Non-resonant contributions"
- Standard Model (SM)-like diagrams
- Involves the trilinear self-coupling of h_{125} – λ_{hhh}
  → **probe of the Higgs potential**
- Large destructive interference between triangle and box diagram
  → **suppression of cross-section in SM**
  → **large changes in di-Higgs cross-section possible from BSM effects in λ_{hhh}**

"Resonant contributions"
- Diagrams involving BSM scalars in s-channel (here generically denoted Φ_i)
→ **collider searches for BSM scalars**
- Involve BSM trilinear scalar couplings λ_{ijk}
→ **probe of Higgs potential in extended scalar sectors**

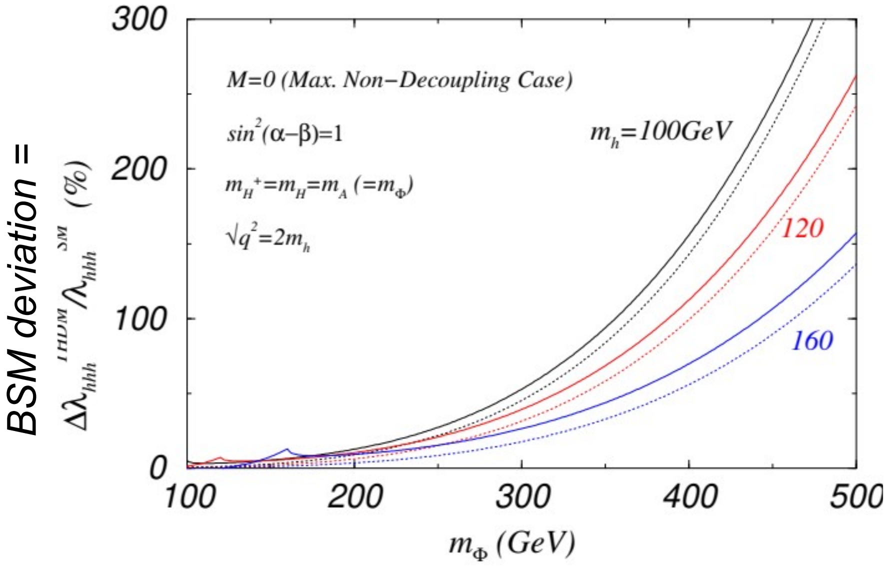
Mass splitting effects in λ_{hhh}

- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM: [Kanemura, (Kiyoura), Okada, Senaha, Yuan '02, '04]



$$\boxed{g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}}$$

$$(\Phi \in \{H, A, H^\pm\})$$



- **Deviations of tens/hundreds of % from SM possible,** for **large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings**
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

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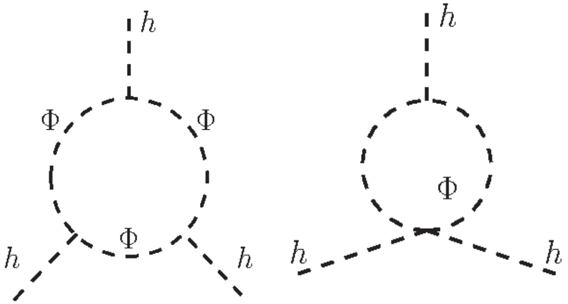

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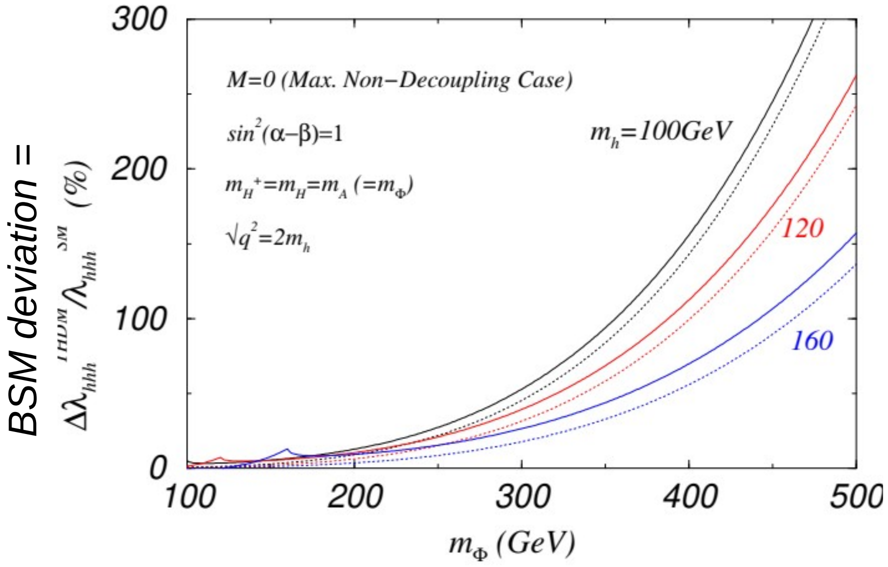

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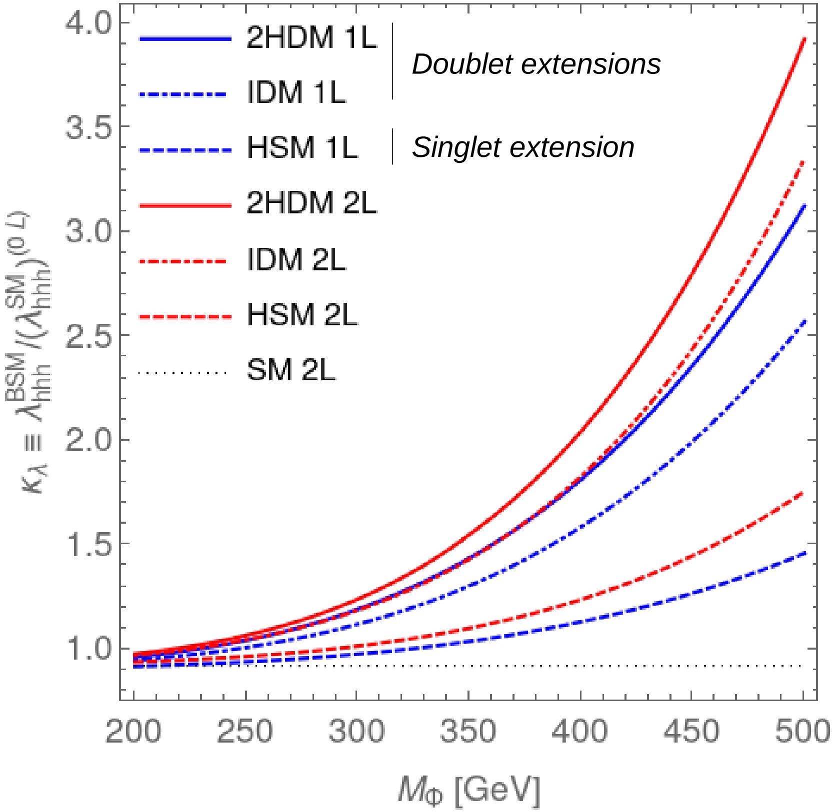
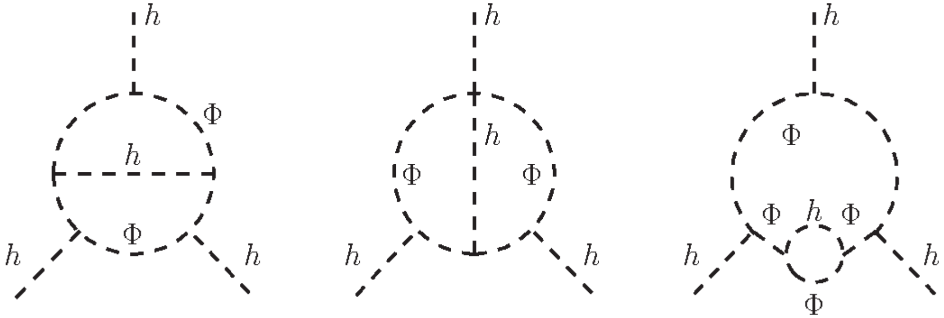
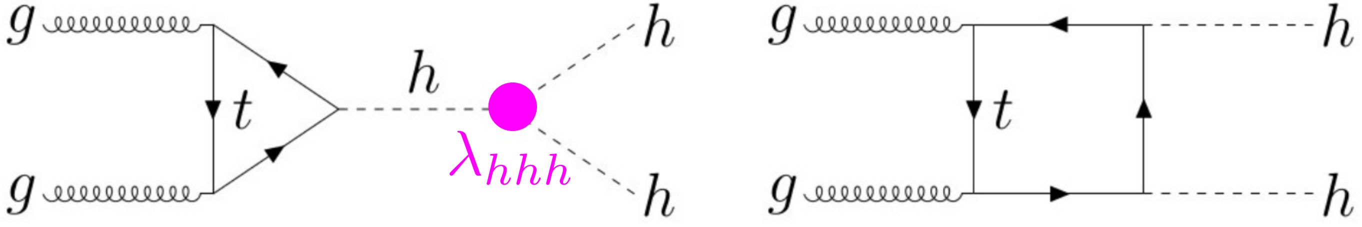
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

- Large effects **confirmed at 2L** in [JB, Kanemura '19] → **leading 2L corrections involving BSM scalars (H,A,H±) and top quark**, computed in effective potential approximation





Interference in non-resonant di-Higgs production

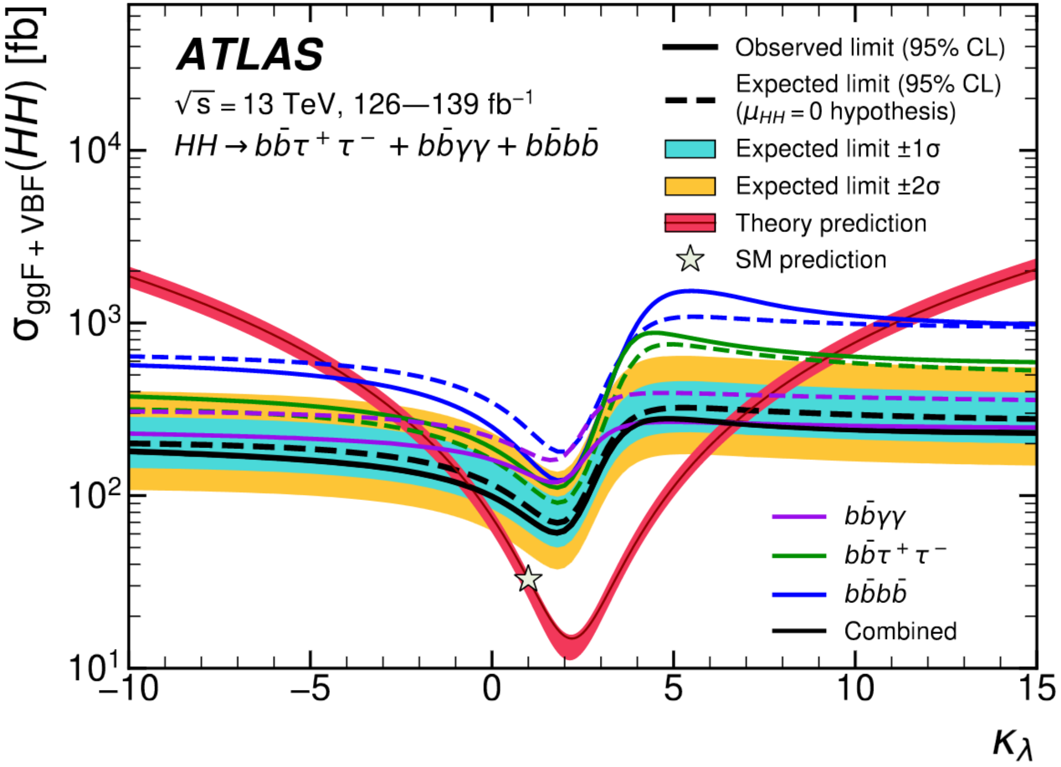


Coupling modifier:

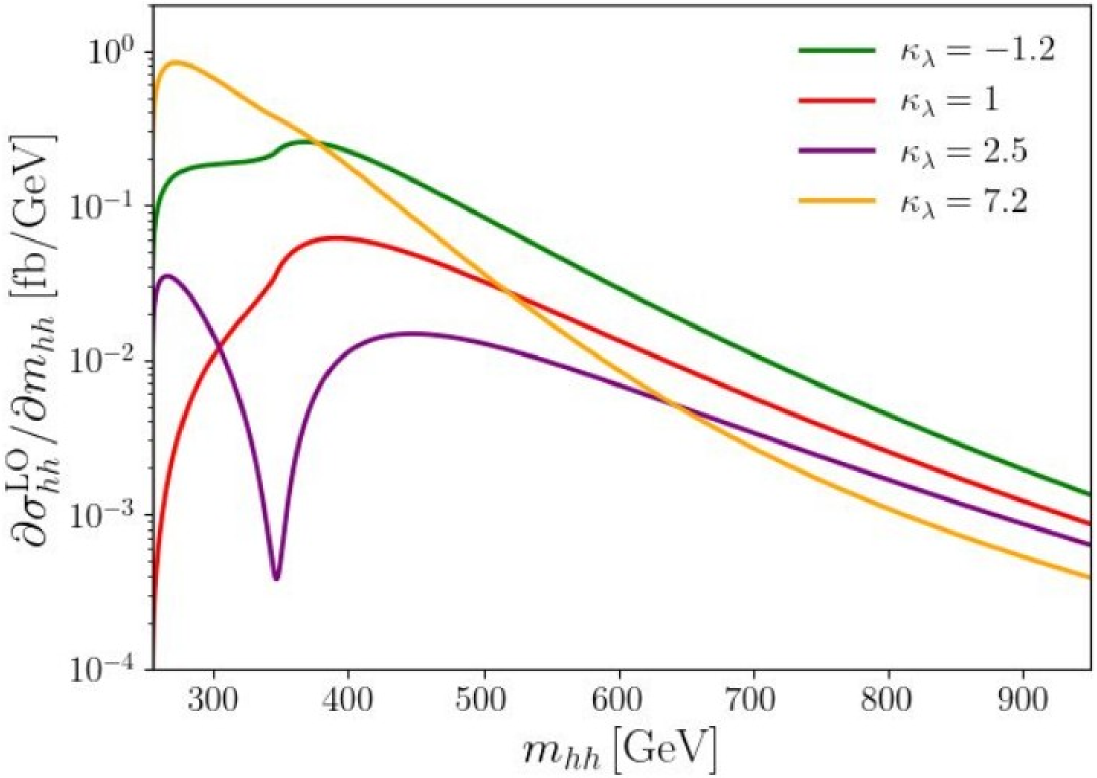
$$\kappa_\lambda \equiv \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\mathrm{SM}}}$$

Relative change in total cross-section for varying κ_λ

Differential m_{hh} distributions for varied κ_λ



[ATLAS 2211.01216]



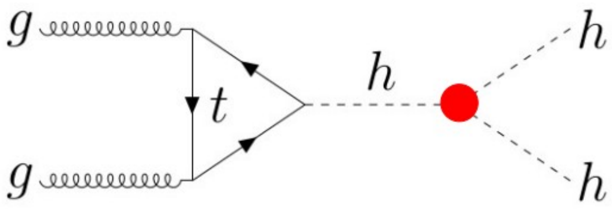
Plots by [K. Radchenko Serdula '24]

Note: impact of change in top Yukawa → overall shift (up/down) of distribution

Di-Higgs production in arbitrary models: `anyHH`

[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

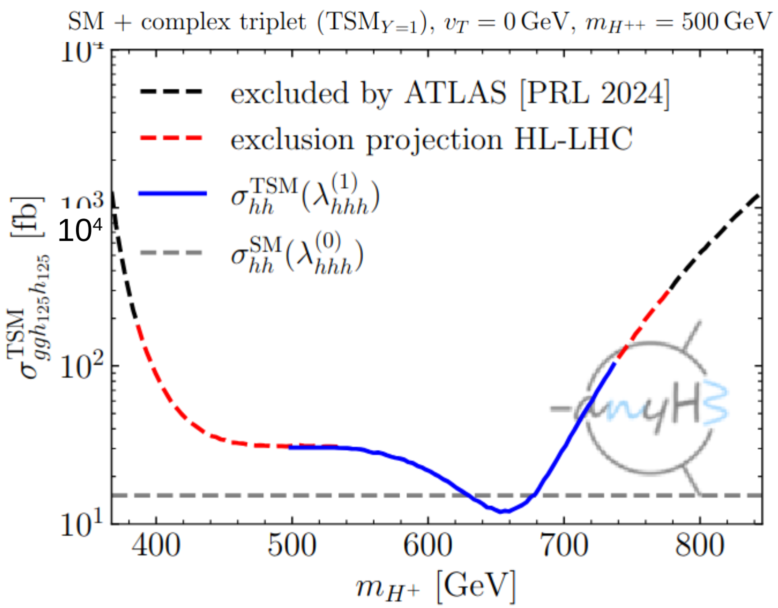
- **`anyHH`: Total and differential cross-sections** (so far, at LO in QCD*) for gg→hh including **1L corrections to λ_{ijk}** (computed by `anyH3` [Bahl, JB, Gabelmann, Weiglein '23]) and **BSM contributions and momentum-dependence in s-channel diagrams**

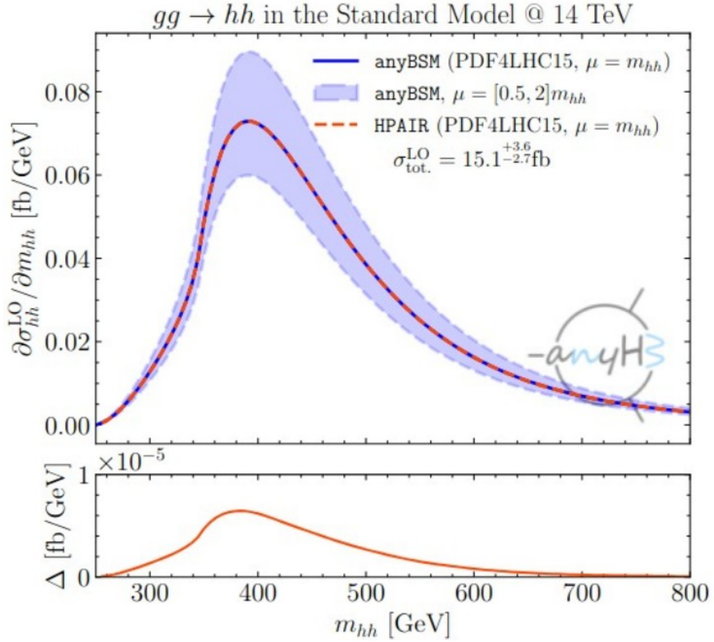


- Takes UFO model files as inputs, as `anyH3`. So far limited to models without additional coloured particles.

- *Here*: example results for the total di-Higgs cross-section in a model with an additional complex triplet.

  Left: total cross-section vs triplet mass in triplet extension of SM
  Right: differential cross-section in SM, compared with HPAIR + with uncertainty band from factorisation scale in PDFs



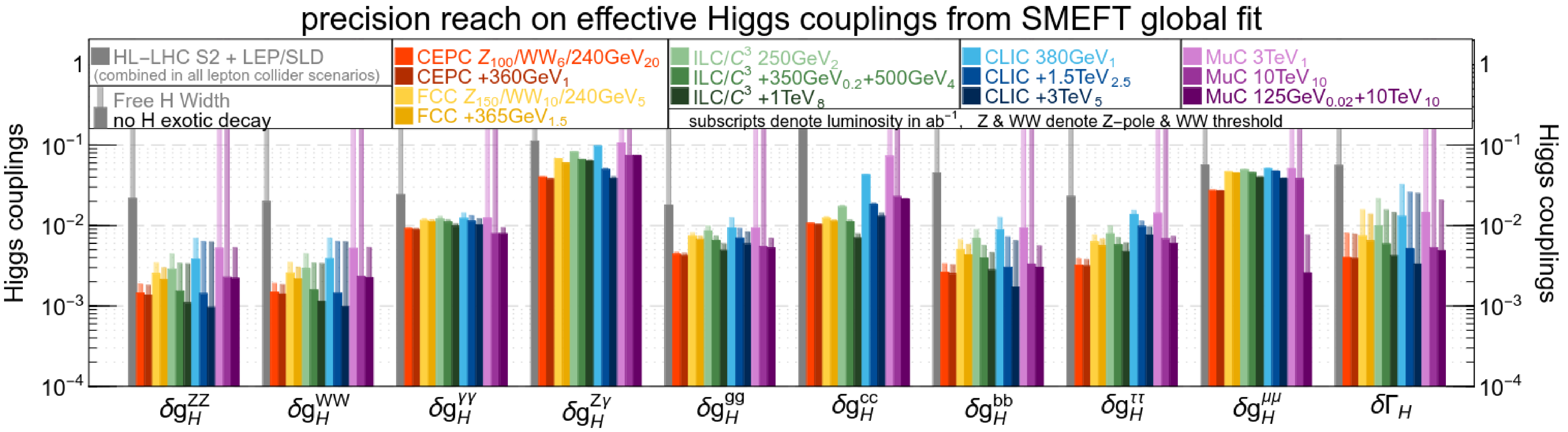


- **Other approach**: calculations in EFTs, e.g. HEFT, including higher-order QCD corrections → see e.g. [Buchalla et al. '18], [Heinrich et al. '20], [Bagnaschi et al. '23]

Future prospects

Future projections for Higgs coupling measurements

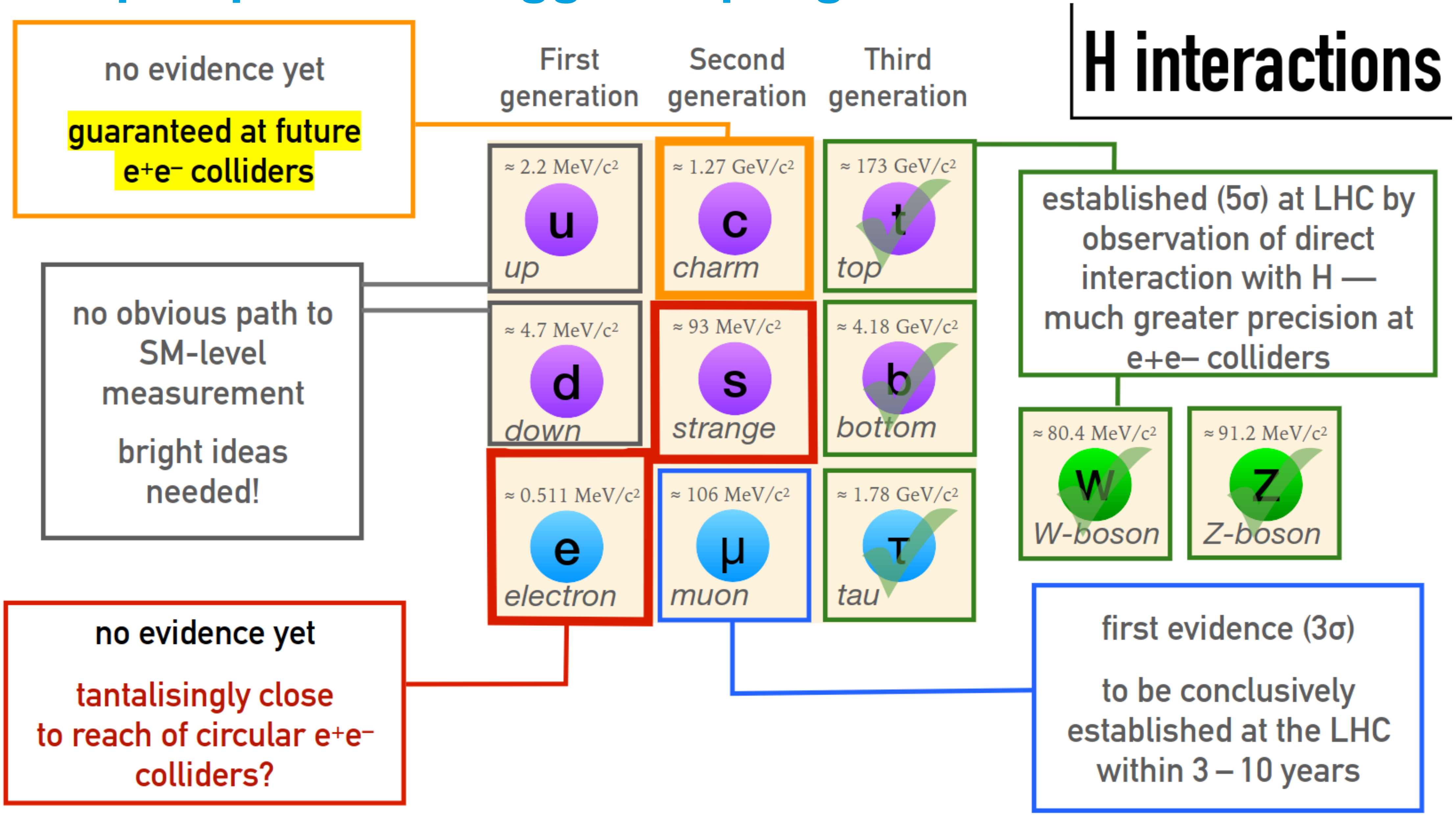
Global fit in SMEFT, using Higgs data,
EW precision observables, di-boson data
e.g. [Snowmass Higgs topical report ‘22]



→ important to **properly assess and compare prospects at future colliders**

→ keep in mind: these numbers also **depend on theoretical uncertainties** (e.g. on calculation of relevant cross-sections) → need to be taken into account and estimated realistically!

Future prospects for Higgs coupling measurements



Slide from [G. Salam '23]

Direct probes of λ_{hhh} at e^+e^- colliders

- **Double-Higgs production, either in $e^+e^-\to Zhh$ or $e^+e^-\to \nu\bar{\nu}hh$**
- Relies however on being above the Zhh threshold!

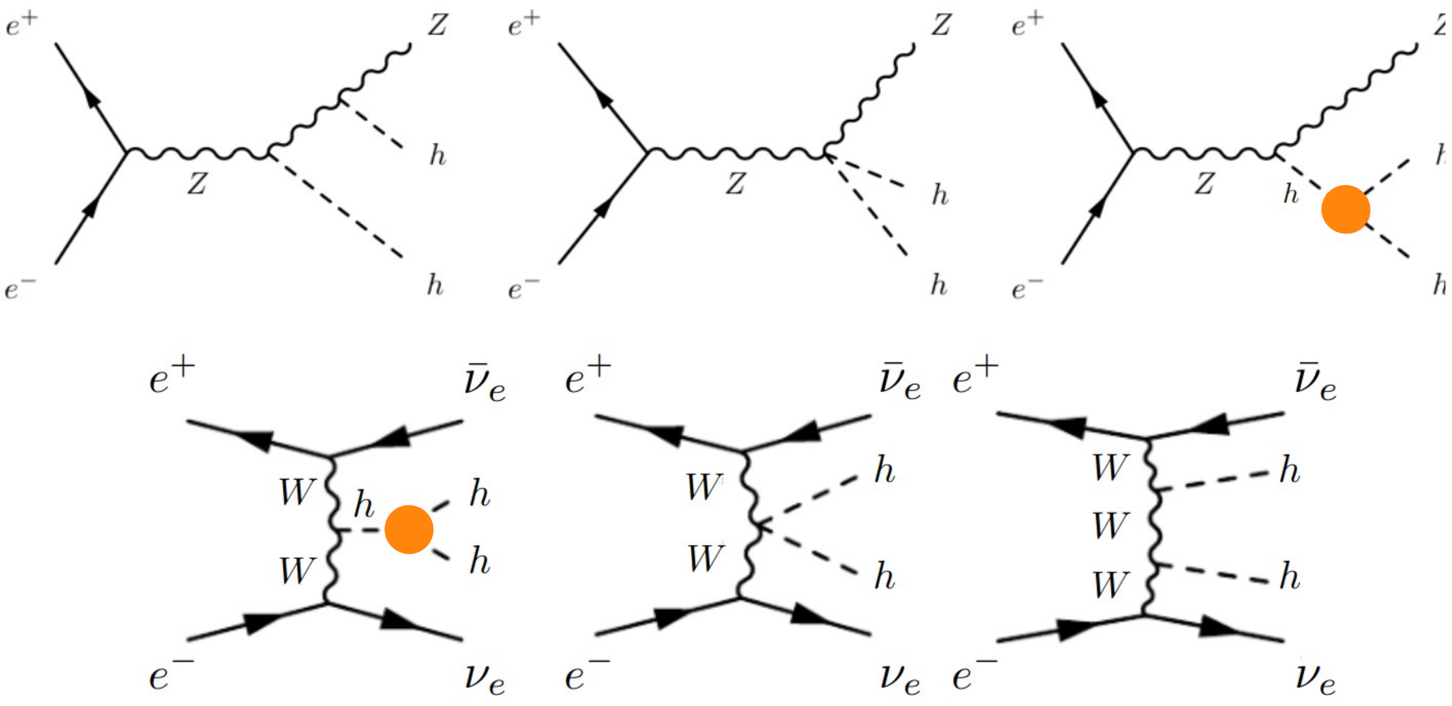
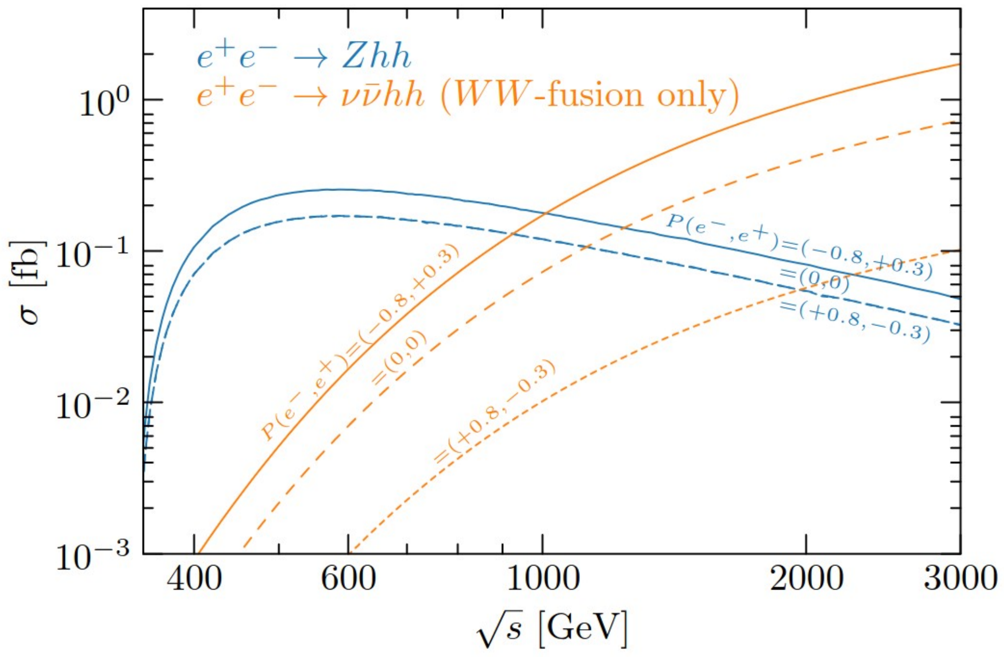


Figure from [De Blas et al. 1905.03764]

Figure from [De Blas et al. 1812.02093]



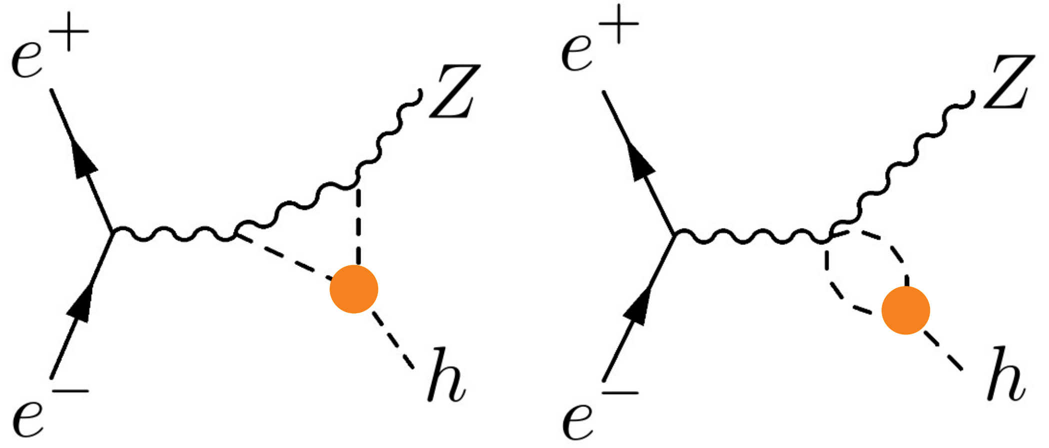
- $e^+e^-\to Zhh$ better at $\sqrt{s}$~500 GeV
- $e^+e^-\to \nu\bar{\nu}hh$ better for larger $\sqrt{s}$

Indirect probes of λ_{hhh} at e^+e^- colliders

- Below the Zhh threshold, λ_{hhh} can still be investigated through its (indirect) effect in quantum corrections to single-Higgs production

- In particular, λ_{hhh} enters NLO corrections to $e^+e^- \to Zh$
  First pointed out in [McCullough '13], numerous works since (also with global analyses in EFT setting)

Figure adapted from [McCullough 1312.3322]



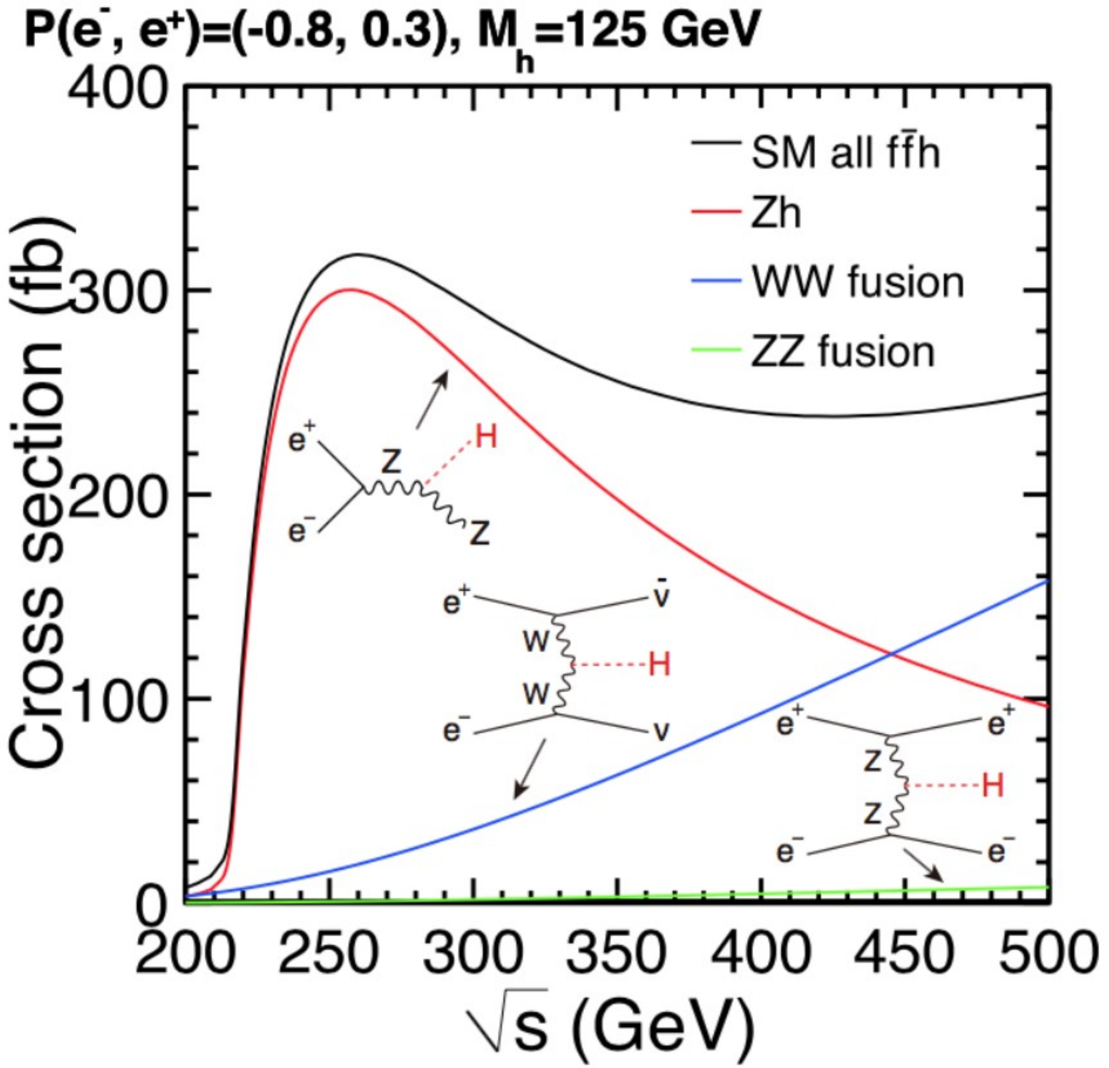


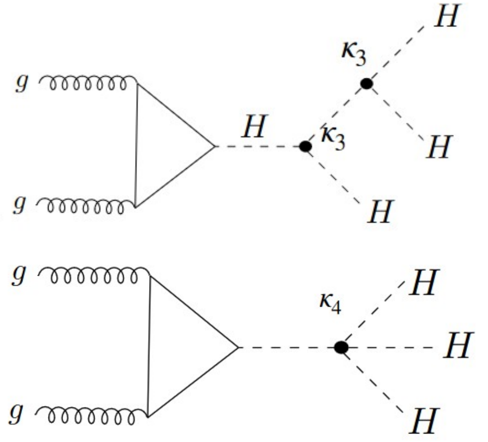
Figure from [Fujii et al. 1710.07621]

- Reliable extraction of λ_{hhh} requires a consistent theory framework and control of $e^+e^- \to Zh$ calculation (including e.g. effects of other BSM operators, etc.) → work in progress

New investigations via triple-Higgs production

Constraining the trilinear and quartic Higgs couplings at the same time

Hadron collider



$\kappa_3 = \kappa_\lambda$: trilinear coupling modifier

κ_4 : quartic coupling modifier

Lepton collider

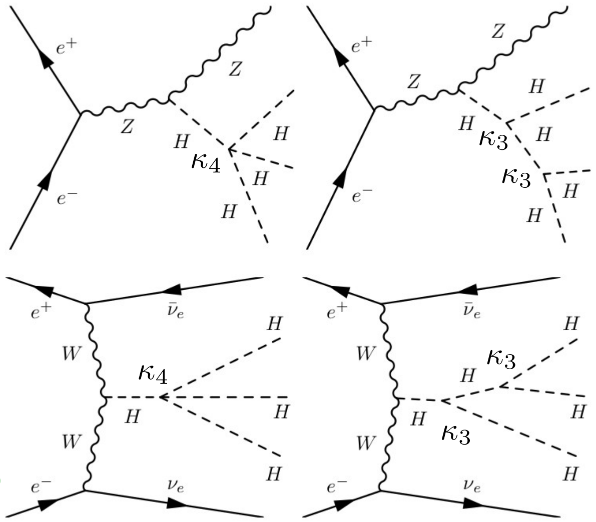
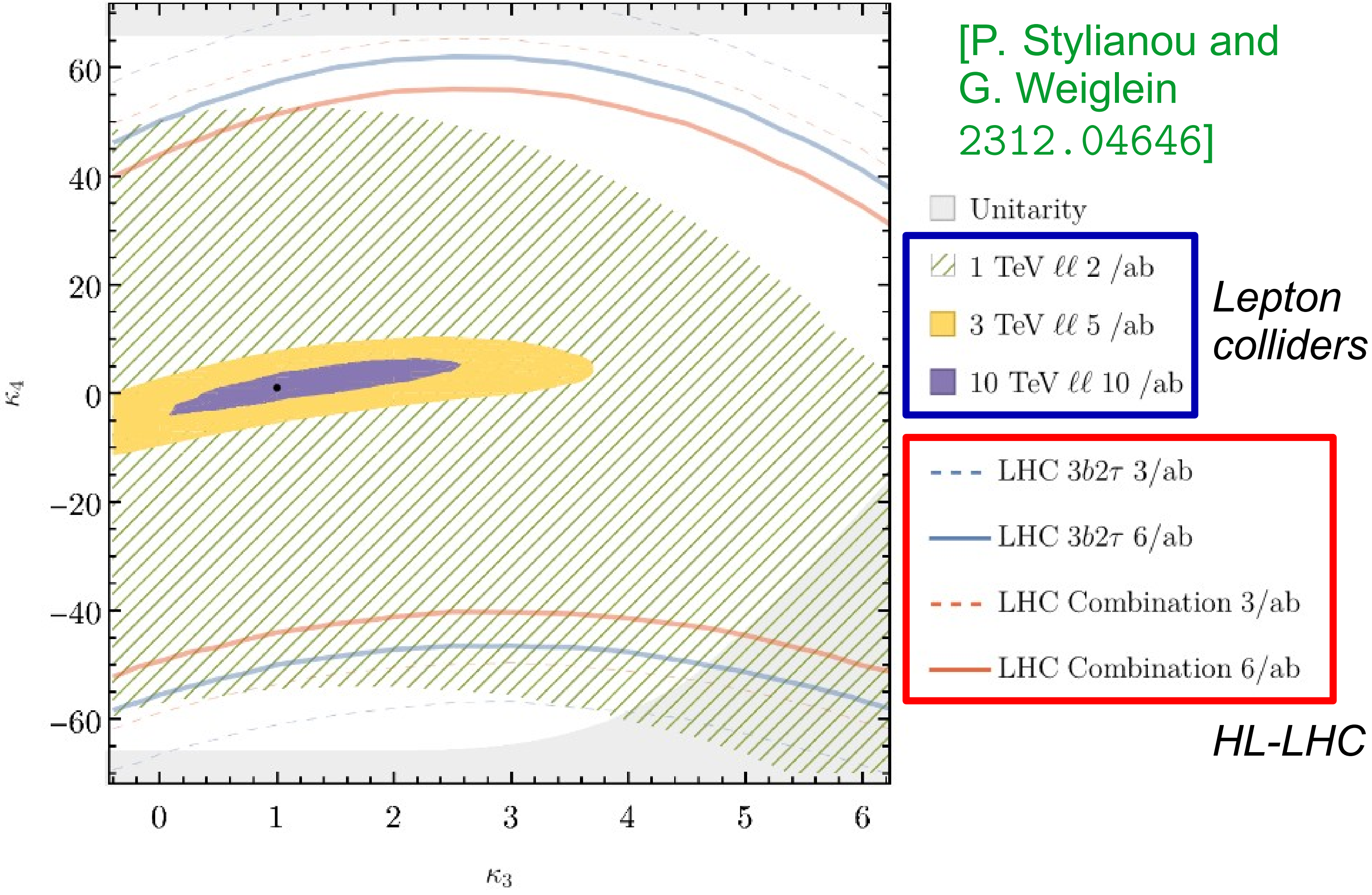


Figure adapted from [Maltoni, Pagani, Zhao 1802.07616]

[P. Stylianou and G. Weiglein 2312.04646]



Summary

- **Detected Higgs boson, h_{125}, plays a central role in investigating the Nature of Physics Beyond the Standard Model**

- Exciting times ahead, with **precision measurements of Higgs boson properties** ongoing at LHC and to be continued at future colliders (HL-LHC, e^+e^- colliders, etc.)

- **High-precision theory predictions are crucial to properly interpret experimental data** in terms of potential discovery, or constraints on the allowed parameter space of New Physics

- Active efforts underway to improve theory calculations in SM and variety of BSM models, with also a push towards automation

Thank you very much for your attention!

Contact

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Addendum 1:
The need for a Higgs boson

Masses of elementary particles

- Strong, weak and electromagnetic fundamental interactions described as **gauge theories**
  - Quantum Chromodynamics (QCD) → $SU(3)_c$
  - Electroweak (EW) interactions → $SU(2)_L \times U(1)_Y$
- Underlying gauge theories is the principle of **gauge invariance**, which strongly constrains allowed terms in the Lagrangian.

  For instance, under a finite local transformation V(x) of a gauge group G, a gauge field A_μ transforms as

$$A_\mu \xrightarrow{V\in G} VAV^{-1} + \frac{i}{g} V(\partial_\mu V^\dagger)$$

  thus a mass term $m_A^2 A_\mu A^\mu$ is **forbidden by gauge invariance**
- Additionally, the currently-known fermions are **chiral**, i.e. weak interactions treat left-handed and right-handed fermions differently → **mass terms for chiral fermions are also forbidden by gauge invariance**

  e.g.

$$m_e \bar{e}_L e_R + \text{h.c.}$$

Y=+1
& part of $SU(2)_L$ doublet

Y=-2
& part of $SU(2)_L$ singlet

Remember:
$$\bar{\psi} = \psi^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- How can we explain the **observed masses** of EW gauge bosons and fermions?

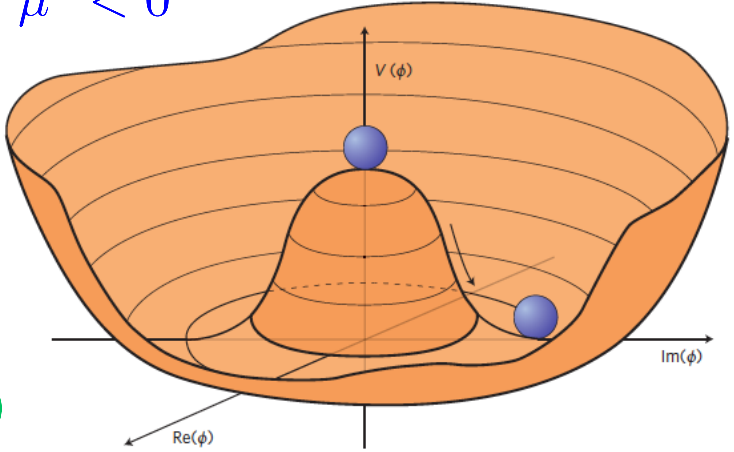
  → **Brout-Englert-Higgs mechanism**

Brout-Englert-Higgs mechanism

- **Idea** (in its minimal realisation): introduce a scalar* Φ – the **Higgs field** – doublet under $SU(2)_L$ and with hypercharge Y=+1, and with potential

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$$

$$\lambda > 0$$
$$\mu^2 < 0$$



* Why a scalar? → so that it can get a vacuum expectation value without breaking Lorentz symmetry

- The potential V(Φ) itself (and thus also the Lagrangian of the theory) obeys the fundamental $SU(2)_L$ x $U(1)_Y$ gauge symmetry but the **vacuum does not**

- In other words, the Higgs field acquires a non-zero **vacuum expectation value v** that triggers the **spontaneous breaking of the EW symmetry (EWSB)**

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \langle\Phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

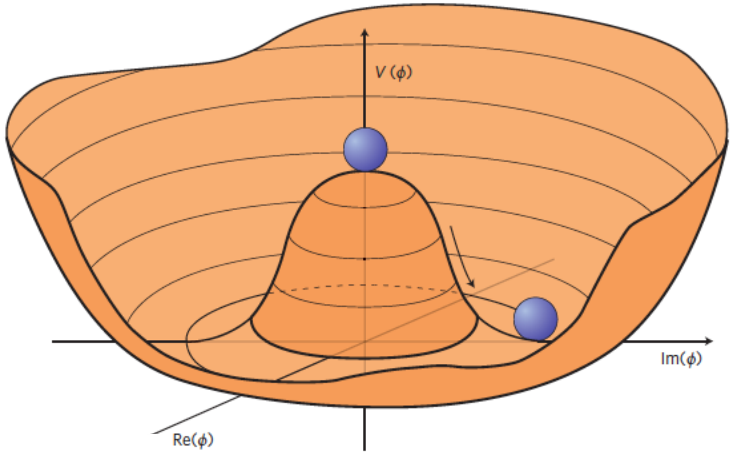
- Vacuum remains symmetric under $\mathbf{U(1)_{QED}}$ gauge group (otherwise there would be charge breaking with strong phenomenological consequences!)

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{EWSB}} U(1)_{\text{QED}}$$

Brout-Englert-Higgs mechanism and particle masses

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4 \qquad \lambda > 0 \quad \mu^2 < 0$$

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{EWSB}} U(1)_{\text{QED}}$$



- Masses of gauge bosons via **scalar kinetic term**, with covariant derivative

$$D_\mu\Phi = \partial_\mu\Phi - \frac{1}{2}i\begin{pmatrix} g_2W_\mu^3 + g_YB_\mu & \sqrt{2}g_2W_\mu^+ \\ \sqrt{2}g_2W_\mu^- & -g_2W_\mu^3 + g_YB_\mu \end{pmatrix}\Phi$$

with $\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}$ *h: Higgs boson* *G^0, $G^\pm$: Goldstone bosons* and $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$

which gives $$|D_\mu\Phi|^2 \supset \frac{1}{4}g_2^2v^2W_\mu^+W^{-\,\mu} + \frac{1}{4}(g_2^2 + g_Y^2)v^2Z_\mu Z^\mu \qquad (\blacktriangle)$$

where $Z_\mu = \dfrac{g_2W_\mu^3 - g_YB_\mu}{\sqrt{g_2^2 + g_Y^2}}$

- **Before EWSB:**
  Φ → 4 degrees of freedom (d.o.f.) + 4 massless gauge bosons of $SU(2)_L$ x $U(1)_Y$ (W_1, W_2, W_3, B) → 4x2=8 d.o.f.

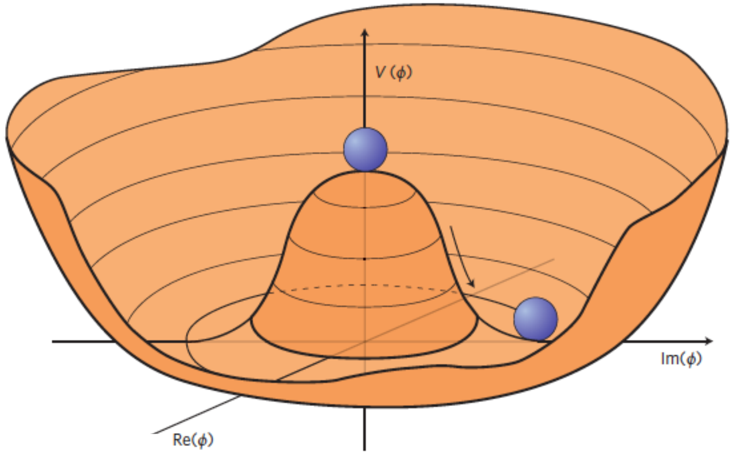
- **After EWSB:** would-be Goldstone bosons are "eaten" by gauge bosons which become massive
  h → 1 d.o.f + 3 massive gauge bosons $W^\pm$, Z → 3x3=9 d.o.f + 1 massless photon A → 2 d.o.f.

  Exercise: rederive equation (▲) + find the expression of the photon A in terms of W_3 and B

Brout-Englert-Higgs mechanism and particle masses

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4 \qquad \lambda > 0 \quad \mu^2 < 0$$

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{EWSB}} U(1)_{\text{QED}}$$



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where $Z_\mu = \dfrac{g_2W_\mu^3 - g_YB_\mu}{\sqrt{g_2^2 + g_Y^2}}$

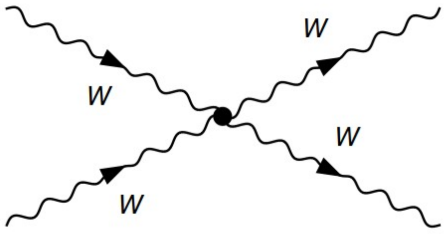
- Masses of fermions (e.g. electron) via **Yukawa-interaction term**

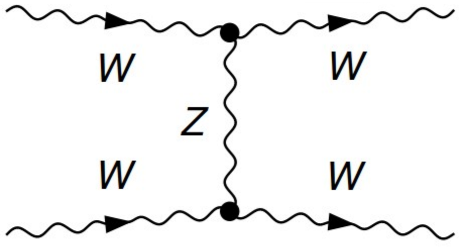
$$\mathcal{L} \supset -y_e\bar{L}_L\Phi e_R + \text{h.c.} \xrightarrow{\text{EWSB}} -\frac{y_e}{\sqrt{2}}v\bar{e}_Le_R + \text{h.c.}$$

Y=+1 conjugate of $SU(2)_L$ doublet — *Y=+1 $SU(2)_L$ doublet* — *Y=-2 $SU(2)_L$ singlet*

Where to find "the" Higgs boson? A unitarity argument

- Higgs-less alternatives to BEH mechanism were also devised (e.g. technicolor)
  → *How to **test** the BEH mechanism? At what scale can the Higgs boson be found?*
- Consider a massive boson W_μ with momentum $k^\mu = (E,0,0,k)$
  → 3 possible polarisations such that $k_\mu \cdot \varepsilon^\mu = 0$ and $\varepsilon_\mu \cdot \varepsilon^\mu = -1$
  → 2 transverse polarisations $\varepsilon_{T1}{}^\mu = (0,1,0,0)$, $\varepsilon_{T2}{}^\mu = (0,0,1,0)$
  + 1 longitudinal polarisation $\varepsilon_L{}^\mu = (k/M_W,0,0,E/M_W) \sim k^\mu/M_W$ for $E \gg M_W$
- Consider the 2→2 scattering of longitudinally polarised W bosons $\mathbf{W_L W_L \to W_L W_L}$
  → *without a Higgs boson*, only gauge-boson diagrams like

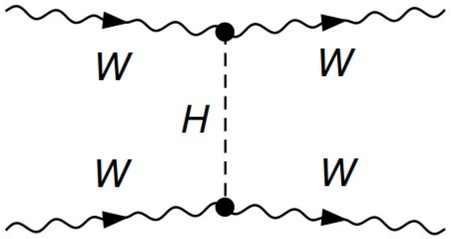




$$\mathcal{A} \sim g_2^2 \frac{E^2}{M_W^2}$$

Loss of unitarity for large E (from ~M_W/g_2)!

→ adding a Higgs boson in the theory:



$$\mathcal{A}_h \sim -g_2^2 \frac{E^2}{M_W^2}$$

$$\Rightarrow \mathcal{A}_{\text{tot}} \sim g_2^2 \frac{M_h^2}{M_W^2}$$

A Higgs boson unitarises the theory if its mass < ~1 TeV

Where to find "the" Higgs boson? A unitarity argument

- Higgs-less alternatives to BEH mechanism were also devised (e.g. technicolor)
  → How to test the BEH mechanism? At what scale can the Higgs boson be found?
- Consider a massive W boson W_μ with momentum $k^\mu = (E,0,0,k)$
  → 3 pos
  → 2 tra
  + 1 lon
- Conside
  → *witho*

No lose theorem (for LHC)

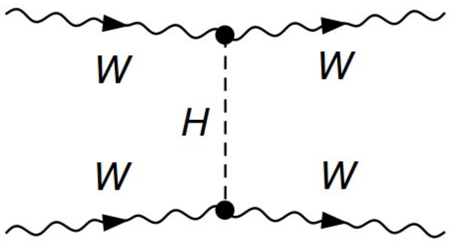
→ **either a Higgs boson exists below/around the TeV scale, to unitarise gauge boson scattering in EW gauge theory**

or

→ **some new strong dynamics would appear at ~ TeV scale**

In other words, theory guaranteed that the LHC would see something!

→ adding a Higgs boson in the theory:



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A Higgs boson unitarises the theory if its mass < ~1 TeV

Addendum 2:
Higgs and BSM

i) Hierarchy problems
ii) Electroweak baryogenesis
iii) Higgs portal to dark matter
iv) Higgs inflation
v) Neutrino mass models with extended Higgs sectors

Naturalness and the gauge hierarchy problem

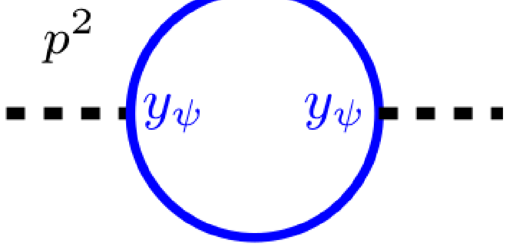
- The EW scale is around m_{EW}~100 GeV (v=246 GeV) while the Planck scale, at which effects of quantum gravity must manifest themselves is M_{Pl}~10^{19} GeV → why are there 17 orders of magnitude between m_{EW} and M_{Pl}?
  → **(gauge) hierarchy problem**

- At a more concrete level, the Higgs mass also poses a technical problem, as it is not ***protected*** from large (quadratic) corrections – unlike for fermions and gauge bosons, nothing forbids scalar mass terms

- Let's consider the effect of a heavy BSM fermion ψ, of mass M ~ M_{pl} with a Lagrangian

$$\mathcal{L} \supset \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - y_\psi\bar{\psi}\psi h$$

  and let's compute the leading corrections to the Higgs mass in this toy model



$$\Delta^{(1\ell)}m_h^2 = -(-iy_\psi)^2 \int \frac{d^dk}{i(2\pi^2)} \mathrm{tr}\left[\frac{i(\not{k}+M)}{k^2-M^2}\frac{i((\not{p}-\not{k})+M)}{(p-k)^2-M^2}\right]$$

$$\approx -\frac{y_\psi^2}{4\pi^2}M_{\mathrm{Pl}}^2 \quad \text{with } p^2 \ll M^2 \quad \& \ Q = M \approx M_{\mathrm{Pl}}$$

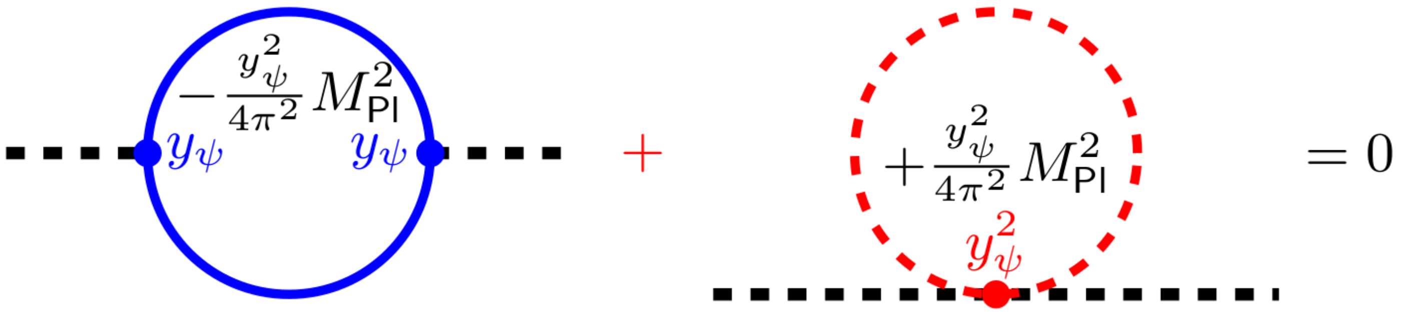
- Getting the Higgs mass right at 125 GeV would imply a tuning between tree-level mass and loop corrections to **32 digits!!!** → **technical hierarchy problem**

Solutions to the gauge hierarchy problem: Supersymmetry

- **Supersymmetry (SUSY)**: [Wess, Zumino ‘74] and many more
  Extend space-time symmetry (Poincaré group) by introducing **new symmetry between fermions and bosons**
  (SUSY is only option to circumvent Coleman-Mandula theorem [Coleman, Mandula ‘67], see [Haag, Lopuszanski, Sohnius ‘75])
  → Each fermion (boson) has a bosonic (fermionic) superpartner, with same mass and related couplings,
  e.g. for toy model of previous slide, ψ has a superpartner $\tilde{\psi}$, with interaction terms

$$\mathcal{L} \supset -y_\psi \bar{\psi}\psi h - \lambda_{\tilde{\psi}} \tilde{\psi}^* \tilde{\psi} h^2 \quad \text{with } \lambda_{\tilde{\psi}} = y_\psi^2, \ m_{\tilde{\psi}} = M_\psi = M_{\mathrm{Pl}}$$

such that



- **SUSY must be broken**, otherwise selectron would have mass 511 keV and would have had to be seen already

- But SUSY can be broken *(super)softly*, i.e. **without reintroducing quadratic divergences in m_h**

$$m_{\tilde{\psi}}^2 = M_\psi^2 + \Delta m^2 \quad \text{and} \quad \lambda_{\tilde{\psi}} = y_\psi^2$$

- Numerous phenomenological models, such as Minimal Supersymmetric Standard Model (MSSM), Next-to-MSSM (NMSSM), Dirac gaugino models, etc., however so far no sign of SUSY at the LHC...

Solutions to the gauge hierarchy problem: Compositeness

- **Compositeness:** see e.g. [Agashe, Contino, Pomarol '04], [Giudice, Grojean, Pomarol, Rattazzi '07] + refs therein
  Light scalars already known in Nature, e.g. pions, but these are *not fundamental*, rather bound – or in other words composite – states
  → Introduce a new strongly coupled sector, with a global symmetry group G, spontaneous broken down to H at a scale f

$$G \xrightarrow{SSB} H \supset SU(2)_L \times U(1)_Y$$

NB: only a part of H is gauged!

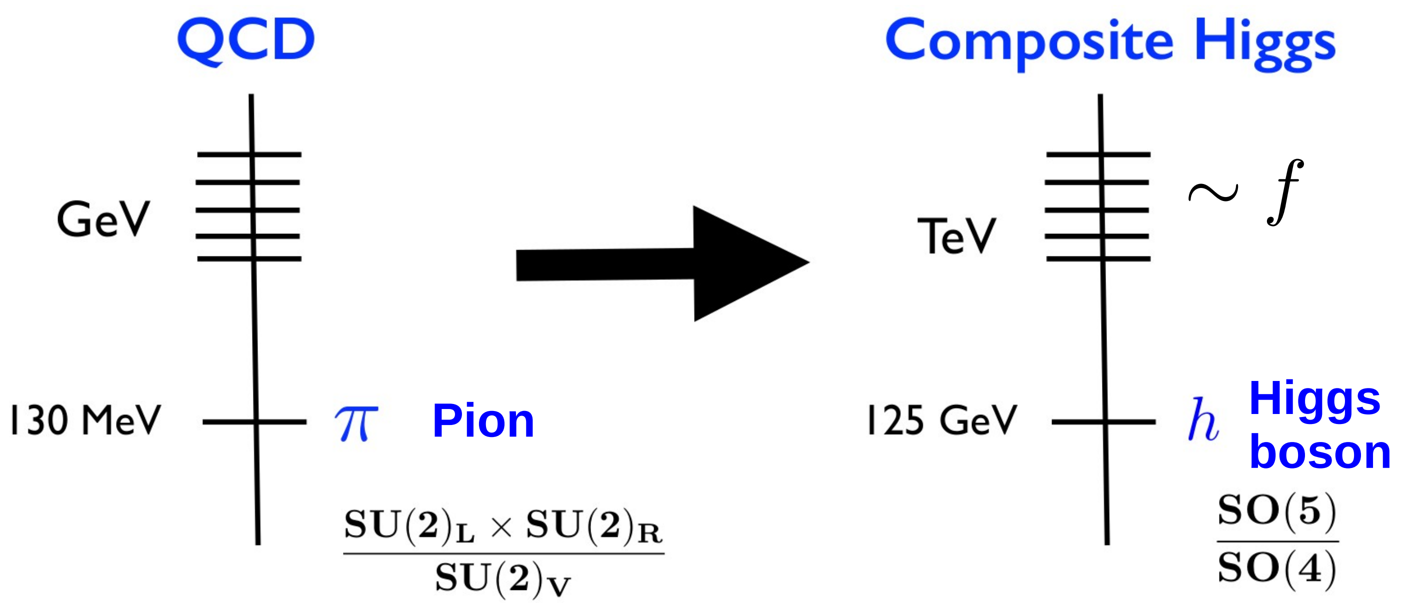
→ **Higgs boson appears as a pseudo-Goldstone boson** → naturally light

Minimal model (1 Higgs doublet):
→ G = SO(5) (10d); H = SO(4) (6d)

Composite Two-Higgs-Doublet Model:
→ G = SO(6) (15d); H = SO(4) x SO(2) (7d)

- Ratio v/f determined by *misalignment* between directions of G/H and $SU(2)_L x U(1)_Y / U(1)_{QED}$ breakings

- *Partial compositeness* to explain quark mass paterns



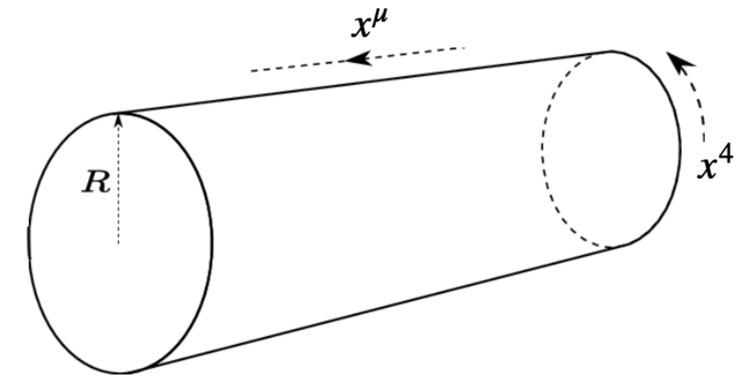
Spontaneous breaking of chiral symmetry in QCD

Minimal Composite Higgs Model

Figure adapted from [Grojean '23]

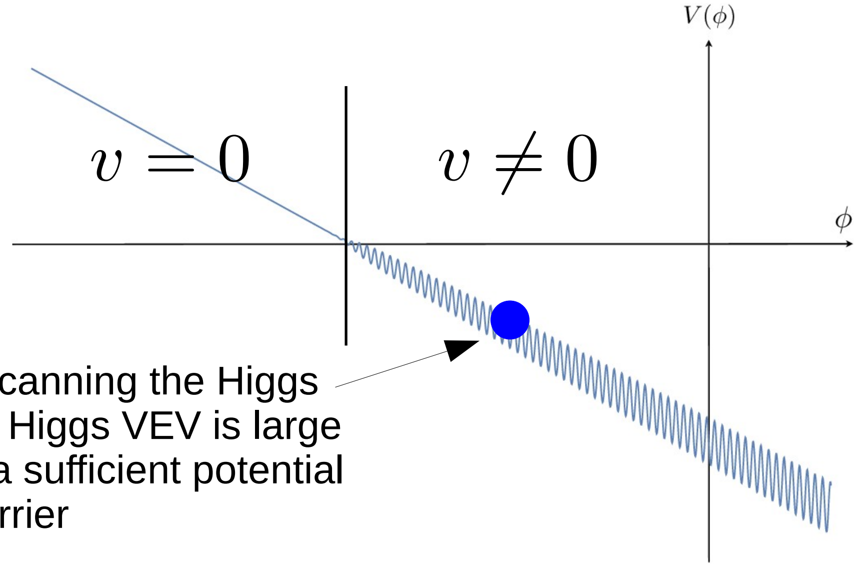
Other solutions to the gauge hierarchy problem

- **Large Extra-dimensions:** [Arkani-Hamed, Dimopoulos, Dvali '98]
  (see e.g. Randall-Sundrum models, [Randall, Sundrum '99])
  Add at least one more dimension of space-time, which is compactified
  → tower of excited states (Kaluza-Klein modes)
  + effective Planck scale in 4d is lowered



- **Gauge-Higgs unification:** [Manton '79], [Fairlie '79], [Hosotani '83], etc.
  Hosotani mechanism: In 5d, a gauge boson contains 5 components
  → 4 components = 4d gauge boson + 1 component = 4d Higgs boson (which triggers EWSB)
  → Higgs mass is then again protected by gauge symmetry in 5d

- **Cosmological relaxation:**
  see e.g. [Graham, Kaplan, Rajendran '15], [Espinosa et al. '15]
  Promote the Higgs mass term μ^2 to a dynamical field,
  the **relaxion**, and give this field a potential and interactions
  with the Higgs boson (and VEV) such that it selects the
  appropriate value of μ^2

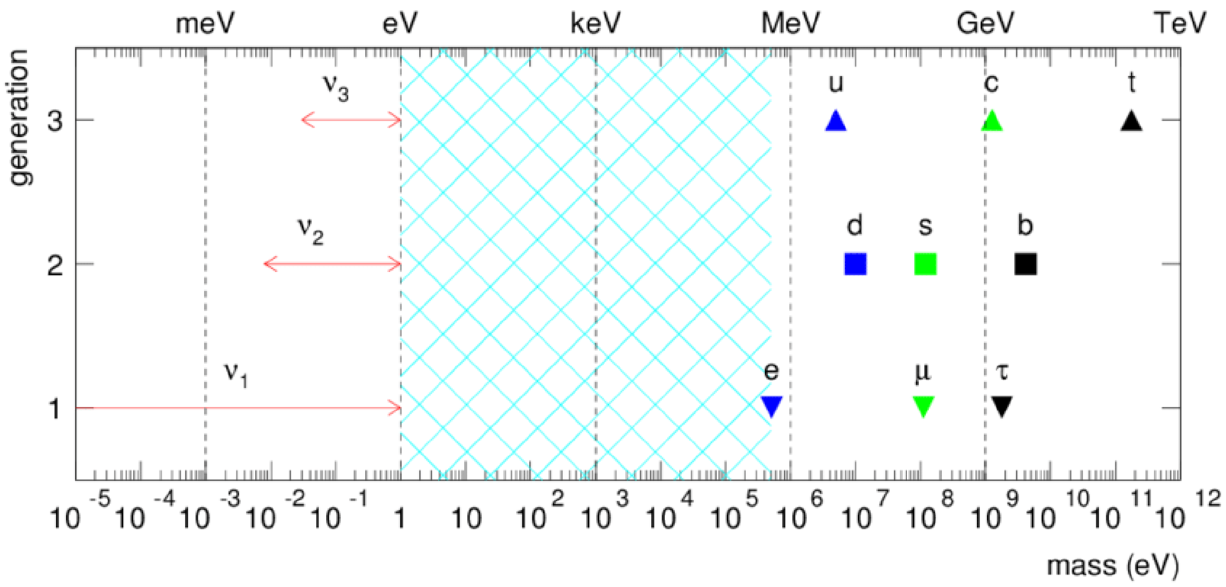


Relaxion stops scanning the Higgs mass μ^2 when the Higgs VEV is large enough to create a sufficient potential barrier

and many more...

The Yukawa hierarchy problem and flavour

- Fermion mass patterns completely unexplained
  why is $m_t \sim 3 \times 10^5\, m_e$? (not to mention neutrinos…)
- Fermion masses in SM → entirely determined by Yukawa couplings between fermions and Higgs boson
  → why does the Higgs treat the three fermion families (identical w.r.t gauge symmetries) so differently?
- No guiding principle in Yukawa interactions in SM
- Gauge symmetries act on all three fermion families in the same way → something must treat the families differently → for instance a "horizontal symmetry" ?



[de Gouvea, hep-ph/0411274]

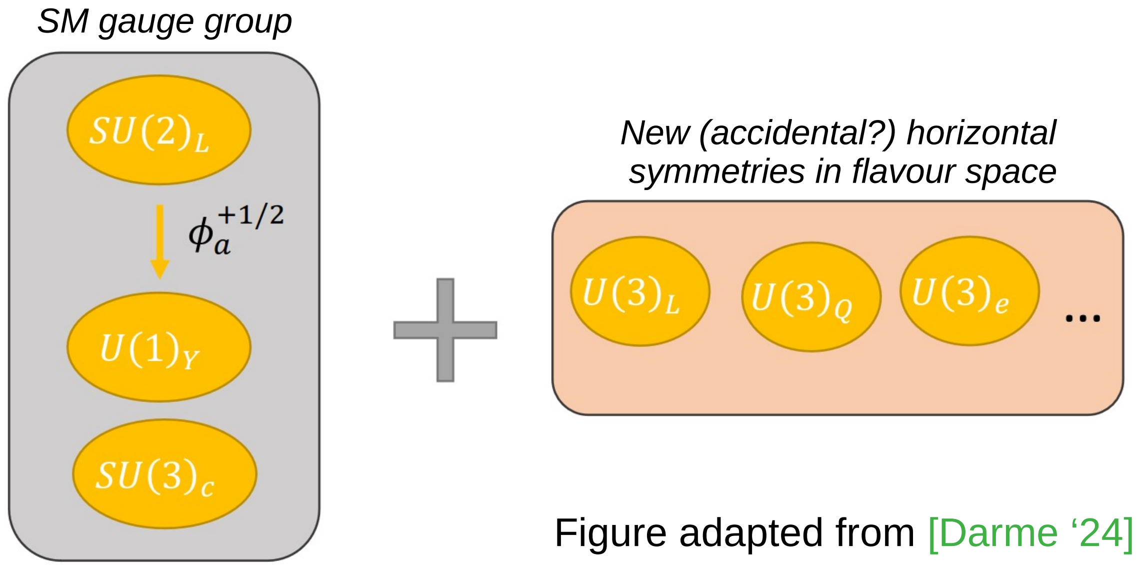
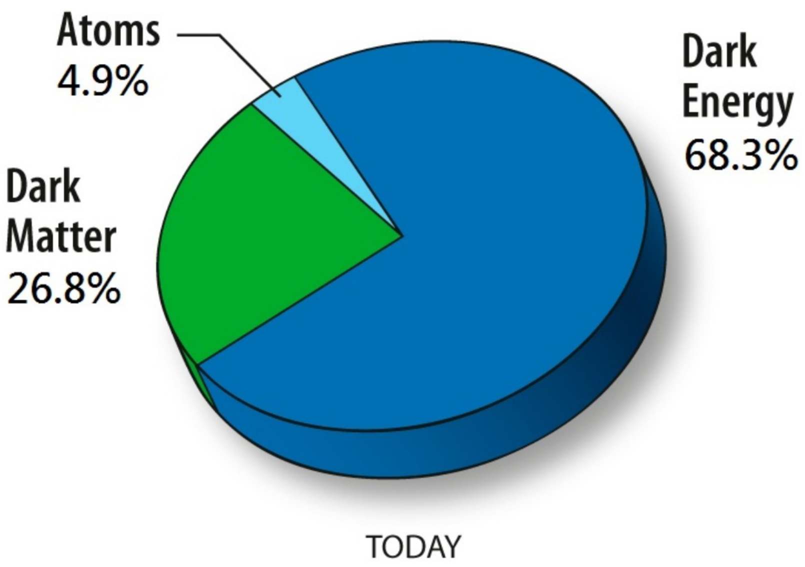


Figure adapted from [Darme '24]

The cosmological constant and its fine-tuning problem



- Cosmological observations → Universe expanding at accelerating pace
- Explained in ΛCDM model by cosmological constant, corresponding to a vacuum energy:

  [Planck '15] $\rho_{vac} \sim 2.5 \times 10^{-47}$ GeV^4

- Value of Higgs potential at EW minimum not fixed by theoretical arguments, nor constrained by colliders

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4 + V_0$$

$$\longrightarrow V_{\rm min} = \frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4 + V_0 \;= -1.2 \times 10^8 \text{ GeV}^4 + V_0 = \rho_{\rm vac} \sim 2.5 \times 10^{-47} \text{ GeV}^4$$

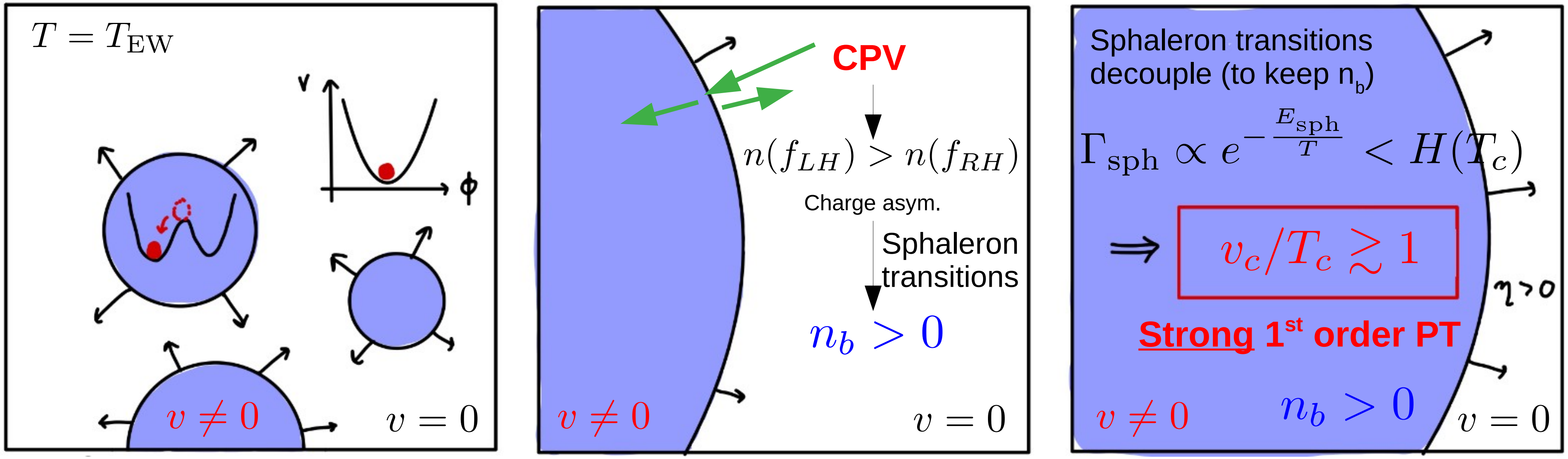
- Cancellation/fine-tuning of **~55 digits** needed in V_0 to reproduce the measured vacuum energy!

  → **cosmological constant problem**

- Possible solutions involve anthropic principle (multiverse), modifications of GR/ΛCDM, or of QFT, etc.

Electroweak Baryogenesis – a brief sketch

- **Sakharov conditions** in EWBG
  1) Baryon number violation → Sphaleron transitions (break B+L)
  2) C and CP violation → C violation + CP violation in extended Higgs sector
  3) Loss of thermal equilibrium → Loss of th. eq. via a strong 1st order EWPT

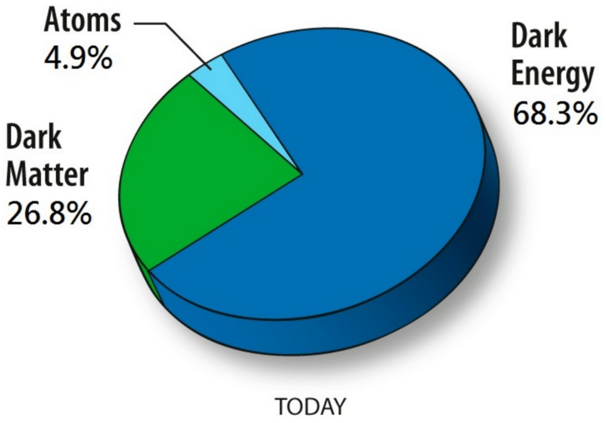


1) Bubble nucleation *2) Baryon number generation* *3) Baryon number conservation*

Figure adapted from [Biermann '22]

- EWBG only involves phenomena around the EW scale → **testable in the foreseeable future** via $\boldsymbol{\lambda_{hhh}}$, collider searches, gravitational waves or primordial black holes (sourced by 1st order EWPT)

Higgs portal to dark sectors



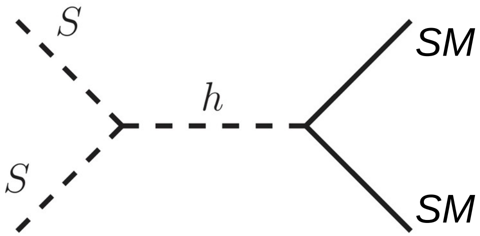
- **Dark matter (DM)**
  - Non-relativistic matter (→ can't be neutrinos)
  - Only/mostly gravitational interactions → several types of astrophysical evidence (e.g. galaxy rotation curves, etc.)
  - Collisionless (c.f. Bullet cluster) & pressureless
  - Needed to seed large-structure formation

  → **No SM particle can fit this!**

- $|\Phi|^2$ is a gauge singlet → Higgs field provides a perfect way to write a **portal term** in the Lagrangian,
  e.g. simplest example = add to SM a singlet S, charged under a global Z_2 symmetry to stabilise DM

$$\mathcal{L}_{\mathbb{Z}_2\mathrm{SSM}} = \mathcal{L}_{\mathrm{SM}} - \lambda_{\mathrm{portal}} S^2|\Phi|^2 - \lambda_{\mathrm{dark}} S^4$$

$\lambda_{\mathrm{portal}}$: *controls DM relic density & detection*



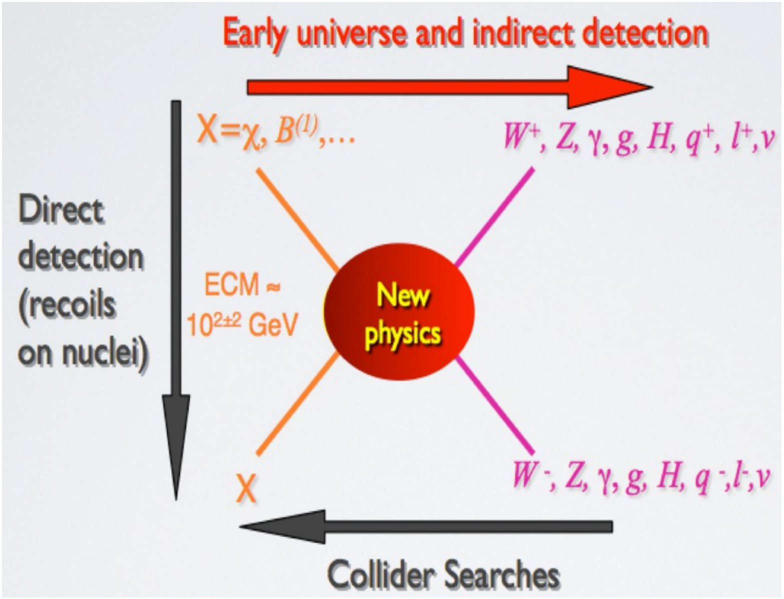
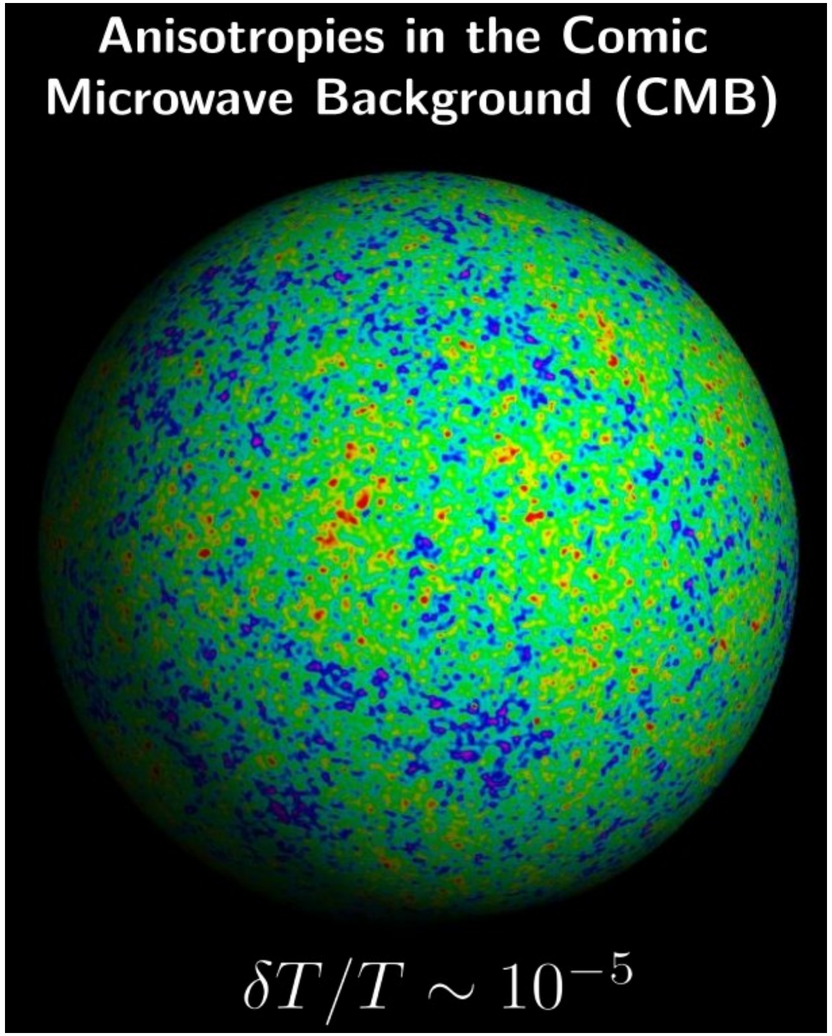


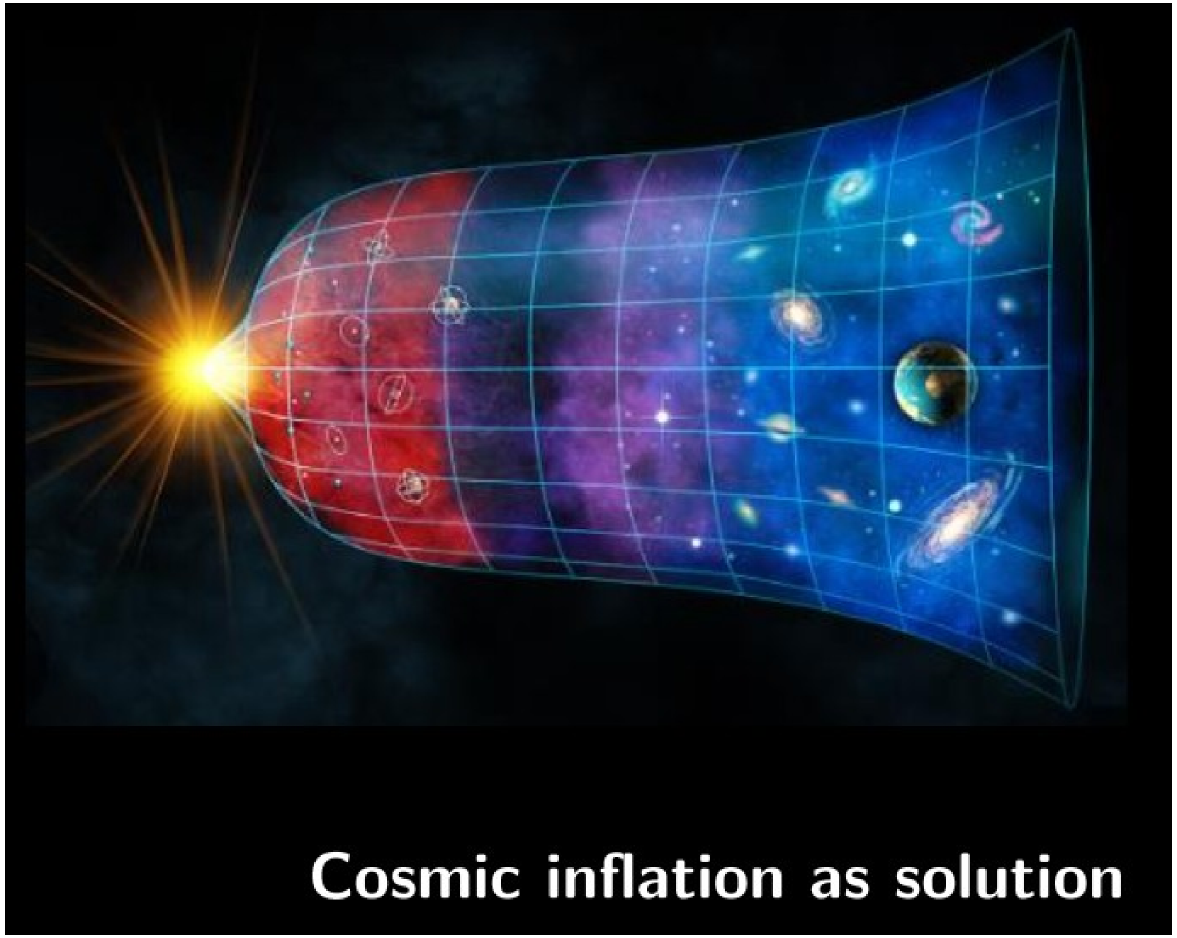
Figure from [Belanger '24]

- Plethora of models: inert singlets, doublets, triplets; Next-to-Two-Higgs-Doublet Model (N2HDM), S2HDM, etc.

Cosmic inflation



[Planck '18]



Figures adapted from [Pierre '23]

The Higgs boson as the inflaton

- Phase of exponential growth driven by scalar field – **inflaton** – with very flat potential → **slow-roll inflation**

- What if the Higgs boson plays the role of the inflaton? [Bezrukov, Shaposhnikov ‘07]
  → **Higgs inflation**
  → Higgs coupled **non-minimally to gravity**

Ricci scalar

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{2}M_{\rm Pl}^2 R - \xi|\Phi|^2 R$$

(in Jordan frame)

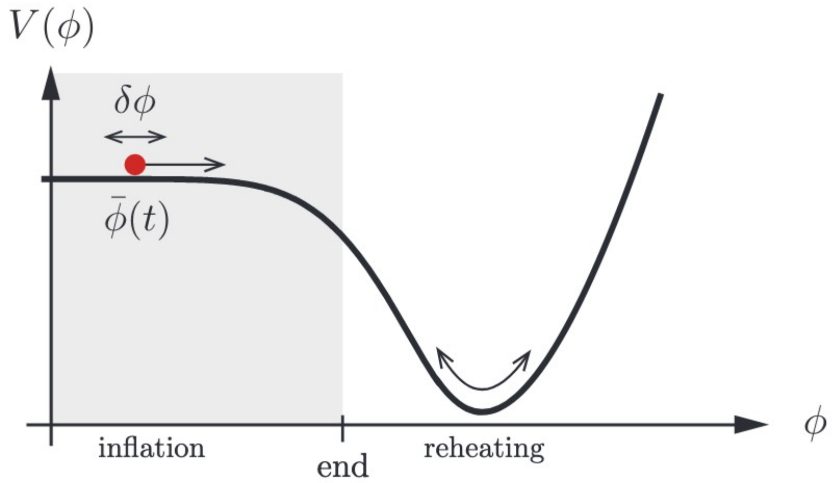
- Change from *Jordan frame* (in which Lagrangian is written) to *Einstein frame* (with canonical coupling to gravity)

$$g^E_{\mu\nu} = \Omega^2(h) g^J_{\mu\nu}, \qquad \text{with } \Omega^2(h) = 1 + \frac{\xi h^2}{M_{\rm Pl}^2}$$

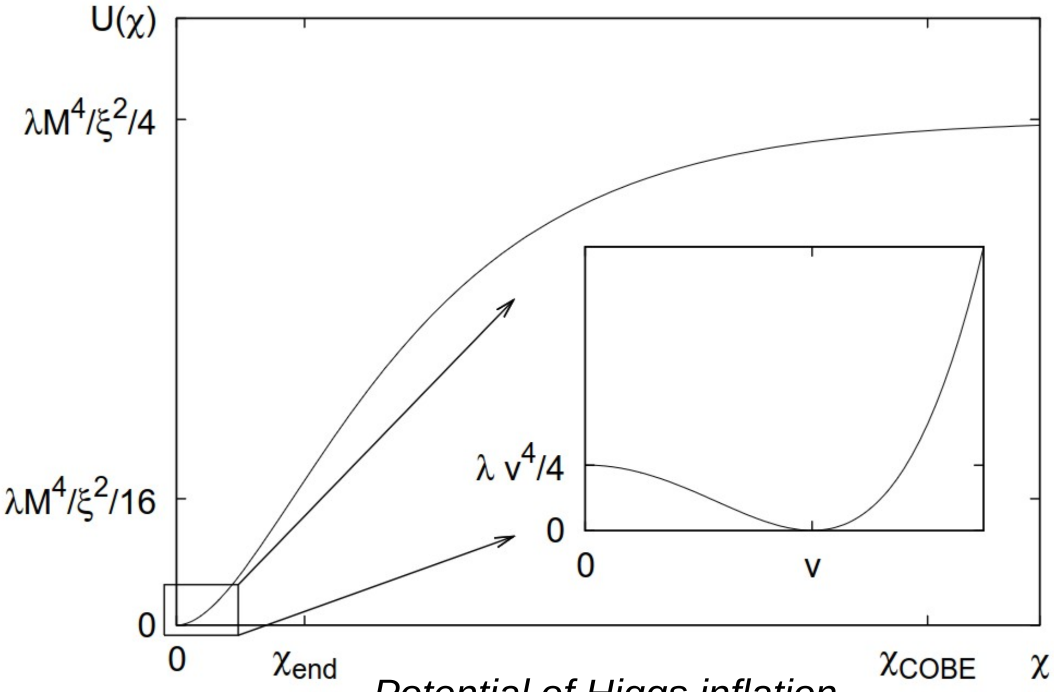
$$\Rightarrow \mathcal{L}^E \supset -\frac{1}{2}M_{\rm Pl}^2 R^E + \frac{1}{2}(\partial_\mu \chi)^2 - \underbrace{\frac{\lambda}{4\Omega^4(h(\chi))}\left(h(\chi)^2 - v^2\right)^2}_{\equiv U(\chi)}$$

χ: Higgs field in Einstein frame

- Numerous developments (non-minimal Higgs sectors, different couplings, etc.)



Usual picture of slow-roll inflation



Potential of Higgs inflation (in Einstein frame)

Figure adapted from [Bezrukov, Shaposhnikov ‘07]

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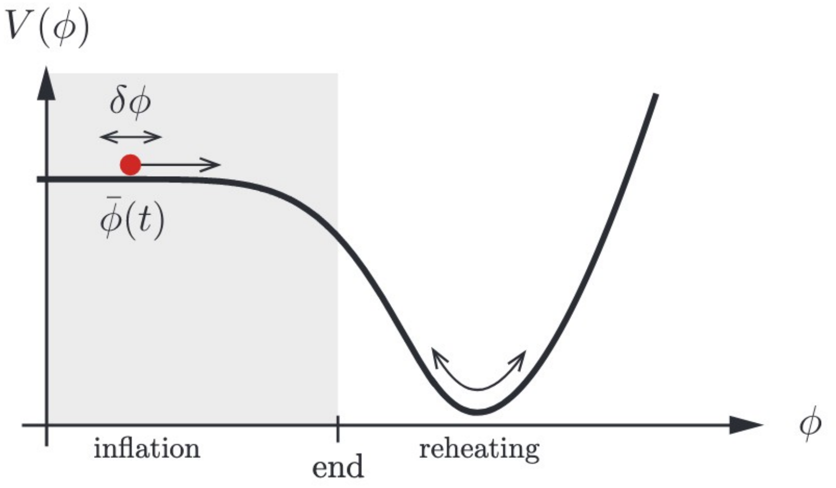

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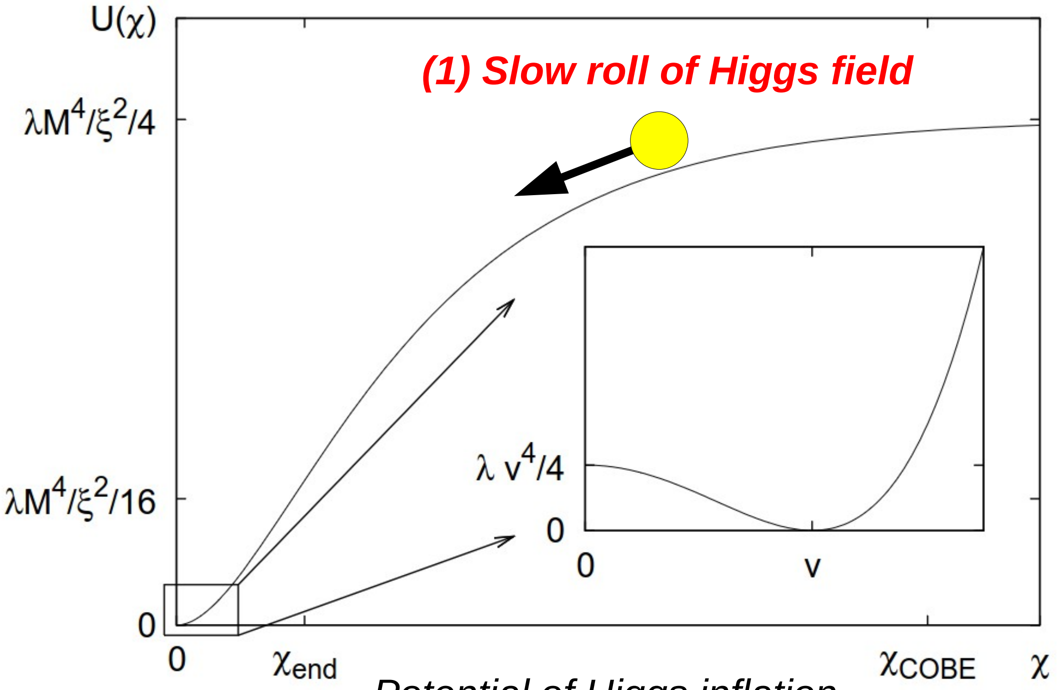

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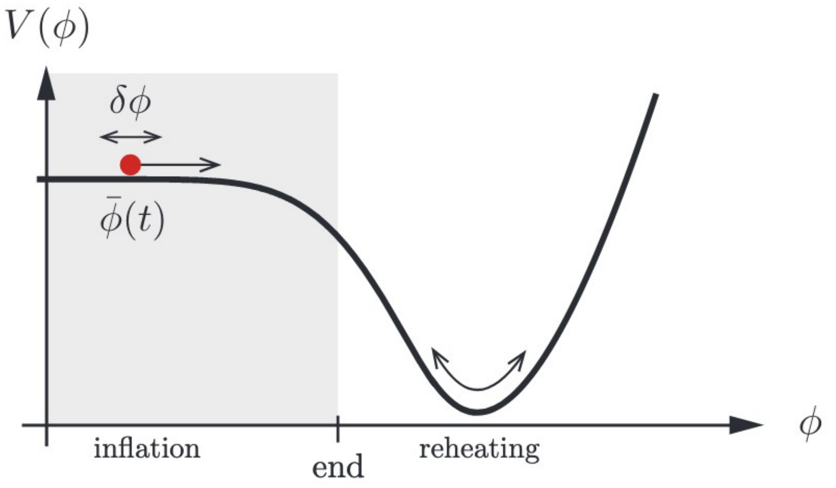

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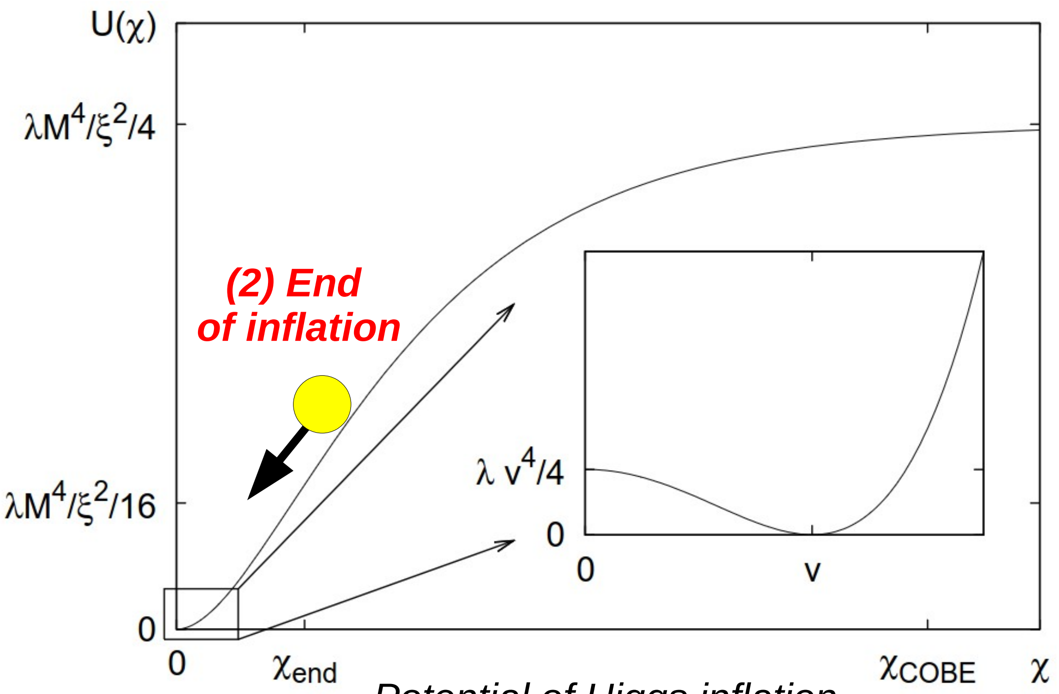

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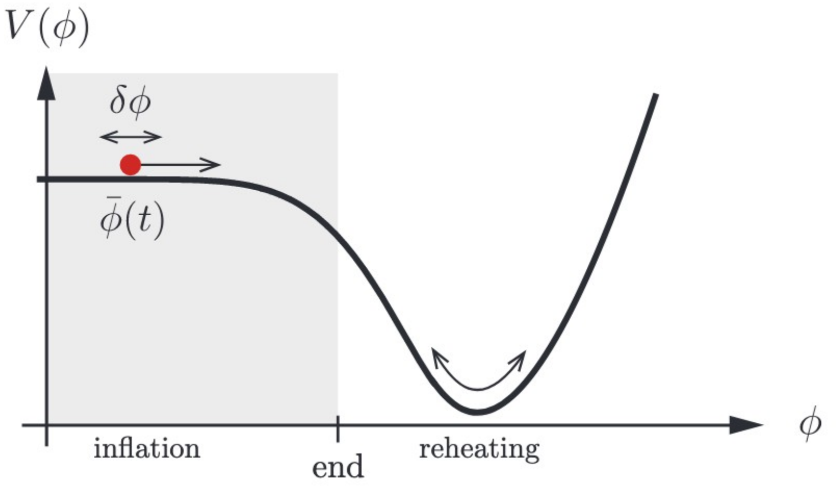

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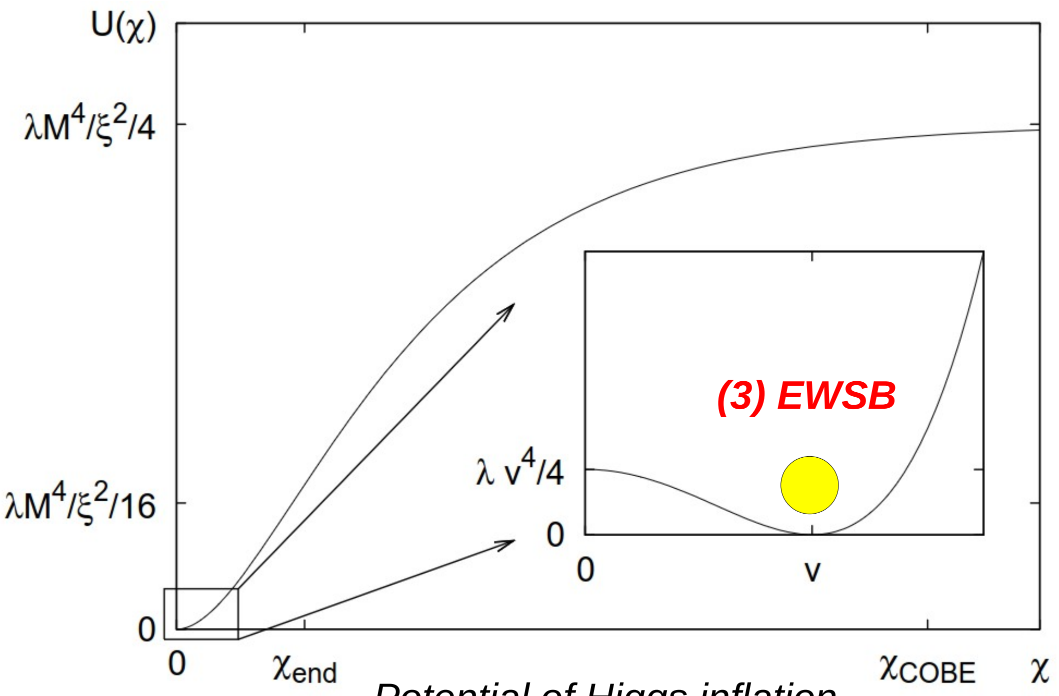


Potential of Higgs inflation (in Einstein frame)

Figure adapted from [Bezrukov, Shaposhnikov '07]

Neutrino masses and Higgs boson(s)

- SM contains **no right-handed neutrinos** → no neutrino masses
- However, since 1960's early signs of neutrino oscillations ("*solar neutrino deficit*"), eventually confirmed ~25 years ago
  → atmospheric neutrino oscillations in 1998
  → solar neutrino oscillations in 2001
  → 2015 Nobel Prize for Kajita and McDonald
  → **neutrinos do have masses** → **extension of SM needed!**

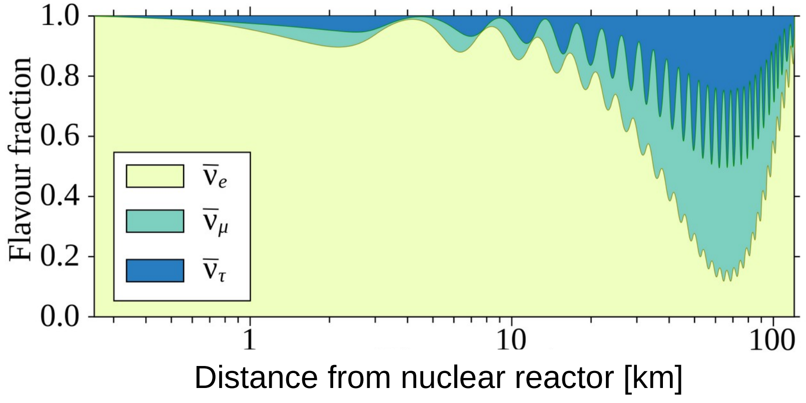


Figure from [Vogel, Wen, Zhang '15]

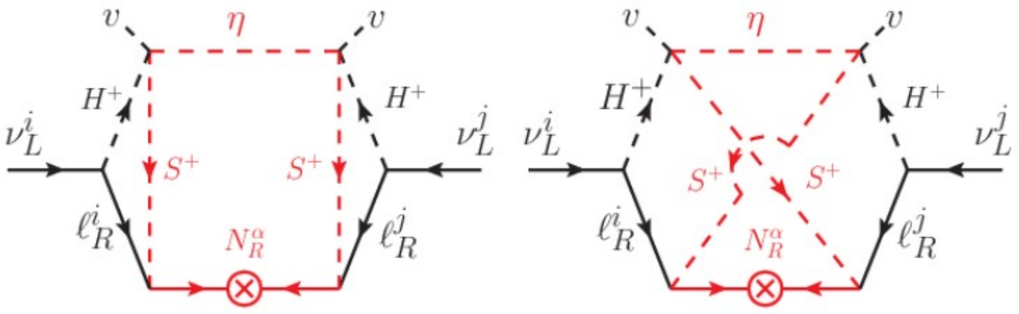
- Most common solutions rely on variants of **seesaw mechanism** (types I, II, III)
  → basic idea (type I): introduce, heavy, right-handed Majorana neutrinos (RHN) N_R

$$\mathcal{L}_{1-fam.} \supset \left(\overline{\nu_L}\ \overline{N_R^c}\right) \begin{pmatrix} 0 & y_\nu v/\sqrt{2} \\ y_\nu v/\sqrt{2} & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} \quad \text{with } M_R \ggg v$$

$$\Rightarrow m_{\nu_L} \sim \frac{y_\nu^2 v^2}{M_R}$$

However, this usually introduces a new hierarchy problem + is difficult to test experimentally

- Other possibility: **generate tiny neutrino masses via radiative effects from <u>extended scalar sectors</u>**
  → [Zee '80], [Babu '88], [Aoki, Kanemura, Seto '08], etc.
  → no longer need for very heavy RHN



*An example of radiative neutrino mass generation: the **Aoki-Kanemura-Seto model***
Figure from [Aoki, Enomoto, Kanemura '22]

Addendum 3:
M_W calculations in the SM and beyond

M_W calculation in the SM I

See e.g. [Awramik, Czakon, Freitas, Weiglein '03], [Hessenberger TUM thesis '18]

- Base for MW calculation is the decay of the muon
  - **Extract G_F from muon lifetime τ_μ** by computing τ_μ in the **Fermi theory**

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(m_e^2/m_\mu^2)\left(1+\frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1+\Delta q)$$

with $F(x) \equiv 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$

Tree-level W propagator contributions (not in Fermi th. but numerically tiny)

QED corrections (known to 1L+2L)

  - Relate M_W, M_Z, α, G_F by **computing muon decay in SM**, and **matching to Fermi theory result**

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

OS scheme

$\Delta r \equiv \Delta r(M_W, M_Z, m_h, m_t, \ldots)$ denotes corrections to muon decay (w/o finite QED effects)

  - Previous relation used to determine M_W as solution, via iterations, of

$$M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

OS scheme

M_W calculation in the SM II

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r) \qquad M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

- At one loop

$$\Delta r^{(1)} = 2\delta^{(1)}Z_e + \frac{\Sigma_{WW}^{(1)}(p^2=0) - \delta^{(1)}M_W^2}{M_W^2} - \frac{\delta^{(1)}s_W^2}{s_W^2} + \{\text{vertex} + \text{box corrections}\}$$

Σ_{WW}: transverse part of the W-boson self-energy, $\delta^{(1)}X$: 1L counterterm to quantity X

- One can show that

$$\delta^{(1)}Z_e \simeq \frac{1}{2}\Delta\alpha + \cdots \quad \text{and} \quad \frac{\delta^{(1)}s_W^2}{s_W^2} \simeq \frac{c_W^2}{s_W^2}\Delta\rho^{(1)}$$

$$\text{with} \quad \Delta\alpha = \frac{\partial}{\partial p^2}\Sigma_{\gamma\gamma}\Big|_{p^2=0} - \frac{\mathrm{Re}\Sigma_{\gamma\gamma}(p^2=M_Z^2)}{M_Z^2}$$

- Leading terms can be rewritten as [Sirlin ‘80]

$$\Delta r^\alpha = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho^{(1)} + \Delta r_{\text{remainder}}(m_h)$$

with $\Delta\alpha$: contribution from light fermion loops to photon vacuum polarisation

$\Delta\rho$: corrections to the ρ parameter

$$\rho \equiv \frac{G_{\text{NC}}}{G_{\text{CC}}} \quad \Rightarrow \quad \rho^{(0)} = \frac{M_W^2}{c_W^2 M_Z^2} = 1 \text{ and } \Delta\rho^{(1)} = \frac{\Sigma_{ZZ}^{(1)}(p^2=0)}{M_Z^2} - \frac{\Sigma_{WW}^{(1)}(p^2=0)}{M_W^2}$$

M_W calculation in the SM III

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r) \qquad M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

- At higher orders

$$\Delta r = \Delta r^{\alpha} + \Delta r^{\alpha\alpha_s} + \Delta r^{\alpha\alpha_s^2} + \Delta r^{\alpha\alpha_s^3 m_t} + \Delta r^{\alpha^2}_{\text{ferm}} + \Delta r^{\alpha^2}_{\text{bos}} + \Delta r^{G_F^2\alpha_s m_t^4} + \Delta r^{G_F^3 m_t^6}$$

QCD (2L+3L+approx.4L) *EW (2L)* *leading 3L corr. to Δρ*

- [Awramik, Czakon, Freitas, Weiglein ‘03] gives a parametrisation as

$$\begin{aligned} M_{\rm W} \;=\; & M_{\rm W}^0 - c_1\,{\rm dH} - c_2\,{\rm dH}^2 + c_3\,{\rm dH}^4 + c_4({\rm dh}-1) - c_5\,{\rm d}\alpha + c_6\,{\rm dt} - c_7\,{\rm dt}^2 \\ & - c_8\,{\rm dH}\,{\rm dt} + c_9\,{\rm dh}\,{\rm dt} - c_{10}\,{\rm d}\alpha_{\rm s} + c_{11}\,{\rm dZ}, \end{aligned}$$

with

$$\mathrm{dH} = \ln\left(\frac{M_{\rm H}}{100\text{ GeV}}\right), \quad \mathrm{dh} = \left(\frac{M_{\rm H}}{100\text{ GeV}}\right)^2, \quad \mathrm{dt} = \left(\frac{m_{\rm t}}{174.3\text{ GeV}}\right)^2 - 1,$$

$$\mathrm{dZ} = \frac{M_{\rm Z}}{91.1875\text{ GeV}} - 1, \quad \mathrm{d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad \mathrm{d}\alpha_{\rm s} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.119} - 1,$$

$$\begin{aligned} M_{\rm W}^0 &= 80.3779\text{ GeV}, & c_1 &= 0.05263\text{ GeV}, & c_2 &= 0.010239\text{ GeV}, \\ c_3 &= 0.000954\text{ GeV}, & c_4 &= -0.000054\text{ GeV}, & c_5 &= 1.077\text{ GeV}, \\ c_6 &= 0.5252\text{ GeV}, & c_7 &= 0.0700\text{ GeV}, & c_8 &= 0.004102\text{ GeV}, \\ c_9 &= 0.000111\text{ GeV}, & c_{10} &= 0.0774\text{ GeV}, & c_{11} &= 115.0\text{ GeV}, \end{aligned}$$

- *Note*: Δr also serves to extract the Higgs VEV from G_F

$$v^2 = \frac{1}{\sqrt{2}G_F}(1+\Delta r)$$

M_W calculation beyond the SM

- Idea of the calculation remains the same, but full theory calculation (that is matched with the Fermi theory one) is now done in the **BSM model**

- In BSM models, M_W ($\leftrightarrow$ muon decay) can receive contributions both at **tree level** and at **loop level**. Considering a model with both sources (and turning to $\overline{\text{MS}}$ for simplicity just here), one can write at 1L [Athron et al. 1710.03760, 2204.05285]

$$M_W^2\big|^{\overline{\text{MS}}} = (M_W^{\text{SM}}|^{\overline{\text{MS}}})^2\left\{1+\frac{s_W^2}{c_W^2-s_W^2}\left[\frac{c_W^2}{s_W^2}(\Delta\rho_{\text{tree}}+\Delta\rho^{\text{BSM}}_{\text{loop}})-\Delta r^{\text{BSM}}_{\text{remainder}}-\Delta\alpha^{\text{BSM}}\right]\right\}$$

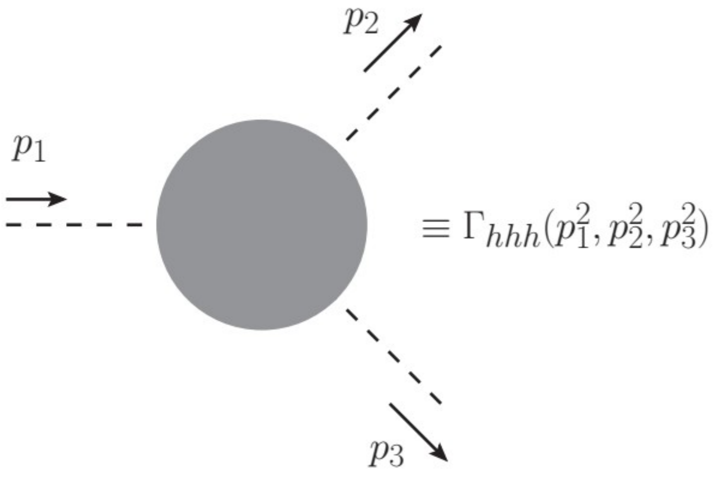
- In the following, we will only discuss models with **$\rho^{(0)}$=1**, and we stay in **OS scheme**

- Some 2L corrections to Δρ known in BSM models
  - $O(\alpha\alpha_S)$ SUSY corrections in [Djouadi et al. '96, '98]
  - $O(\alpha_t^2,\alpha_t\alpha_b,\alpha_b^2)$ in MSSM in [Heinemeyer, Weiglein '02], [Hastier, Heinemeyer, Stöckinger, Weiglein '05]
  - BSM scalar + top quark corrections in (aligned) 2HDM and IDM [Hessenberger, Hollik '16]
- Inclusion of known higher-order SM corrections crucial $\Delta r = \Delta r^{\text{SM}} + \Delta r^{\text{BSM}}$
- Calculations of M_W with Δr to full BSM 1L + partial BSM 2L (from resummation and Δρ) + SM up to 4L
  - MSSM [Heinemeyer, Hollik, Weiglein, Zeune '13]
  - NMSSM [Stål, Weiglein, Zeune '15]
  - MRSSM [Diessner, Weiglein '19]
  - **2HDM & IDM** [Hessenberger '18] (TUM thesis and code `THDM_EWPOS`)

Addendum 4:
Calculations of λ_{hhh}

An effective Higgs trilinear coupling

- In principle: consider 3-point function Γ_{hhh}
  but this is momentum dependent → very difficult beyond one loop



- Instead, consider an **effective trilinear coupling**

$$\lambda_{hhh} \equiv \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min}}$$

entering the coupling modifier

$$\kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\text{SM}}} \qquad \text{with } (\lambda_{hhh}^{(0)})^{\text{SM}} = \frac{3m_h^2}{v}$$

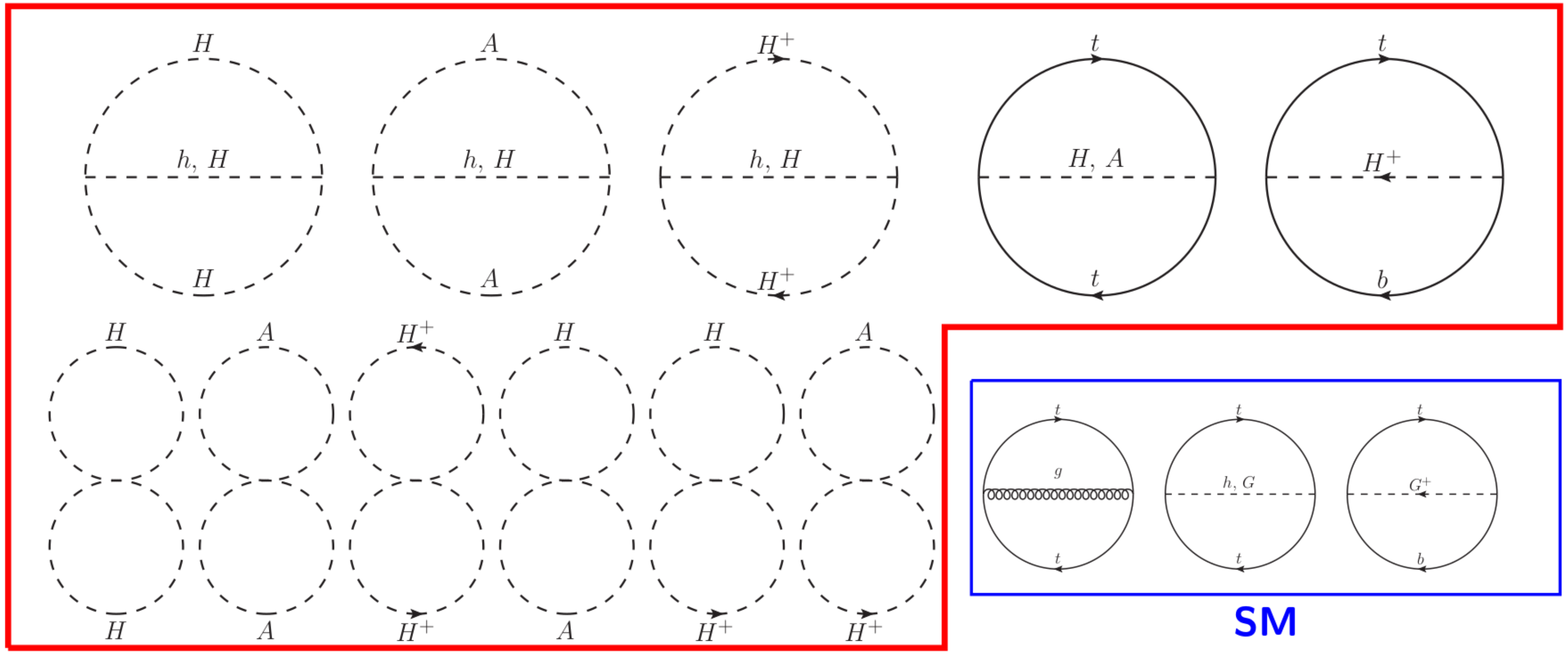
constrained by experiments (*applicability of this assumption discussed later*)

Effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}$ ($\overline{\text{MS}}$ result)
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to* $V^{(2)}$ = diagrams involving **heavy BSM scalars and top quark**
  - **Neglect masses of light states** (SM-like Higgs, light fermions, ...)

2HDM



SM

Effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)}$ ($\overline{\text{MS}}$ result)
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to* $V^{(2)}$ = diagrams involving **heavy BSM scalars and top quark**

- **Step 2**: derive an effective trilinear coupling

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$

($\overline{\text{MS}}$ result too)

Express tree-level result in terms of effective-potential Higgs mass

Effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}$ ($\overline{\text{MS}}$ result)
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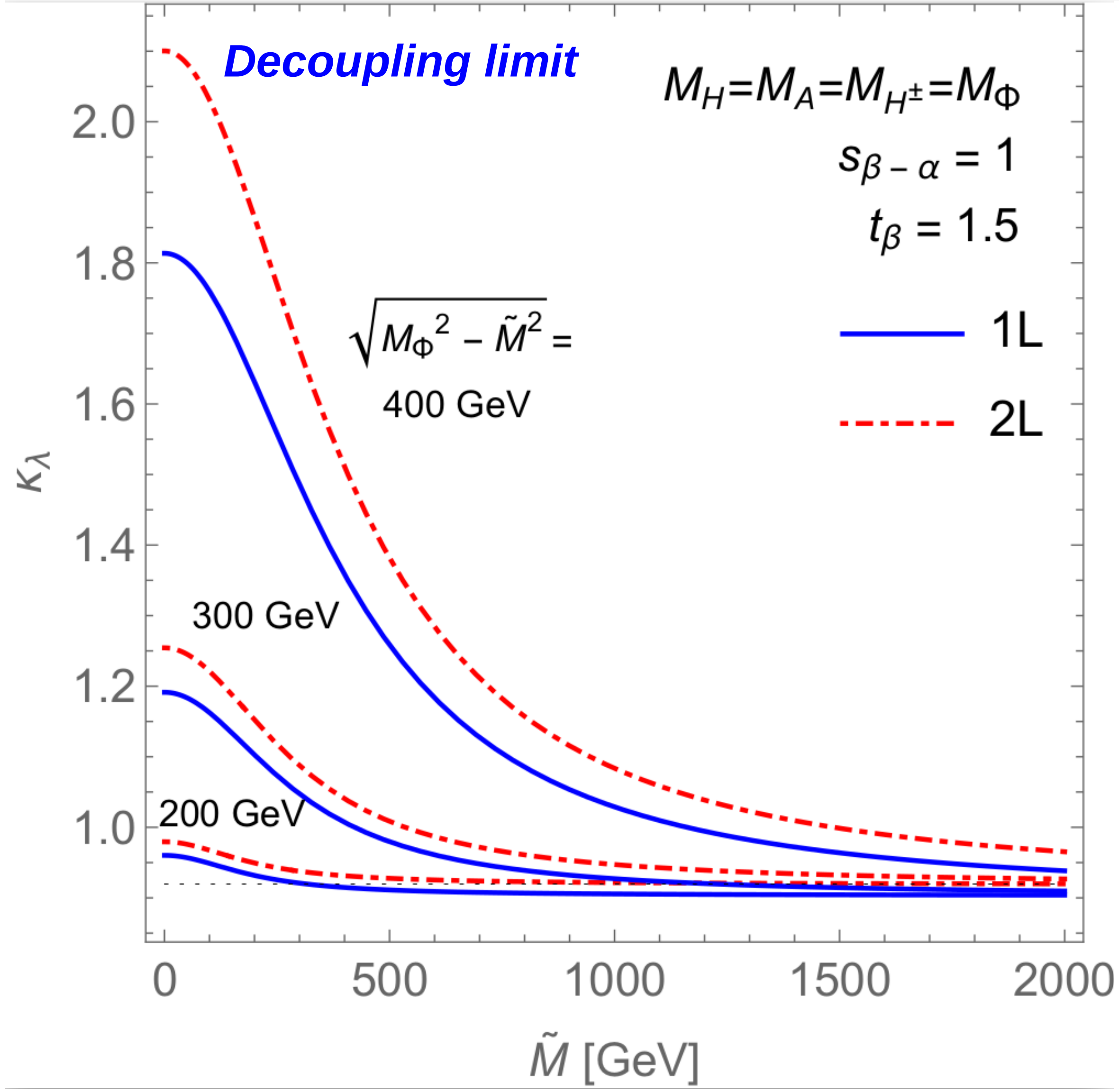
- **Step 2**: $$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$
  ($\overline{\text{MS}}$ result too)

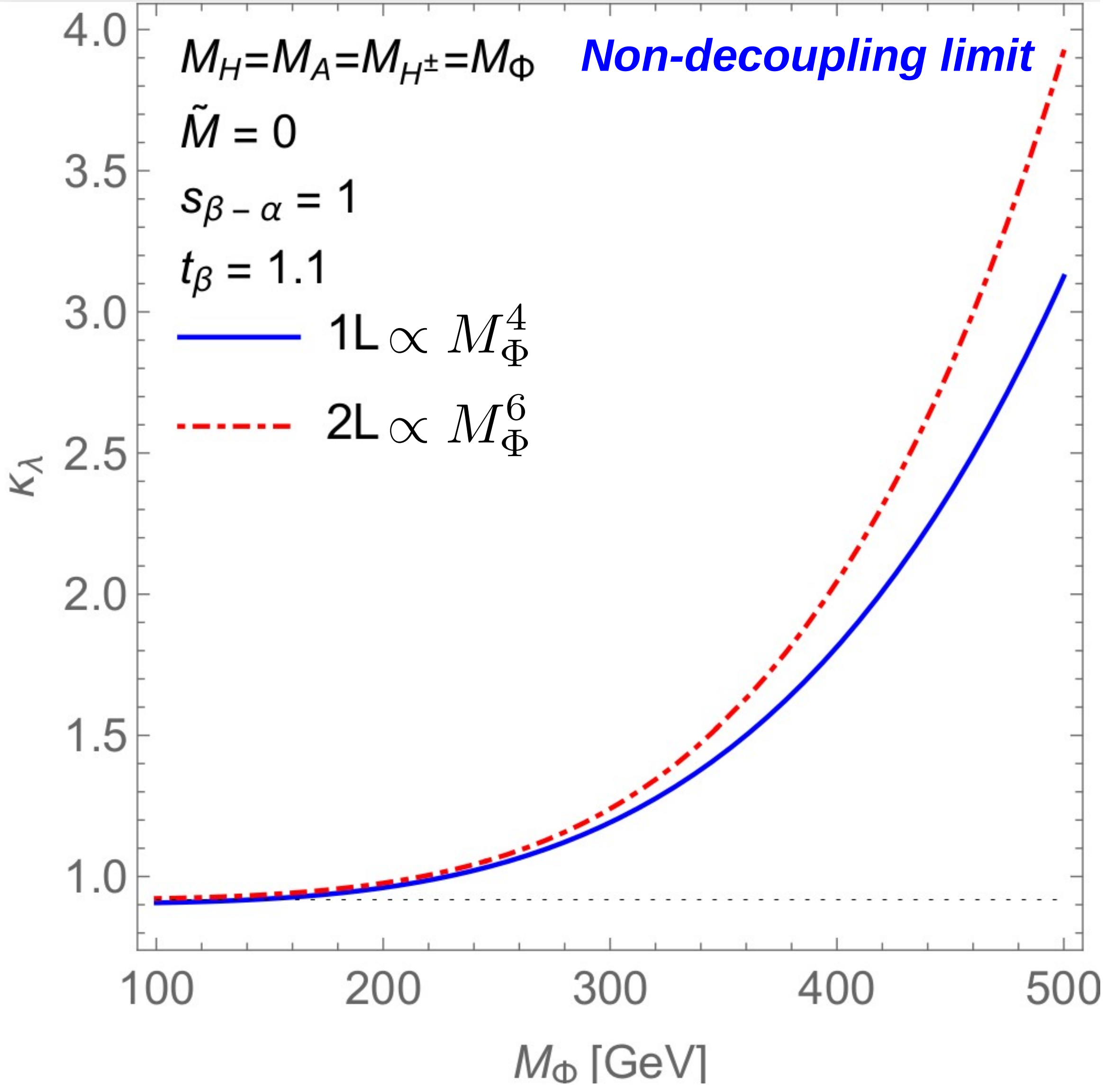
- **Step 3**: conversion from $\overline{\text{MS}}$ to OS scheme
  - Express result in terms of **pole masses**: M_t, M_h, M_Φ (Φ=H,A,H$^\pm$); OS Higgs VEV $v_{\text{phys}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$
  - Include **finite WFR**: $\hat{\lambda}_{hhh} = (Z_h^{\text{OS}}/Z_h^{\overline{\text{MS}}})^{3/2}\lambda_{hhh}$
  - Prescription for M to ensure **proper decoupling** with $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

Our results in the aligned 2HDM

[JB, Kanemura '19]

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_{H^\pm}$



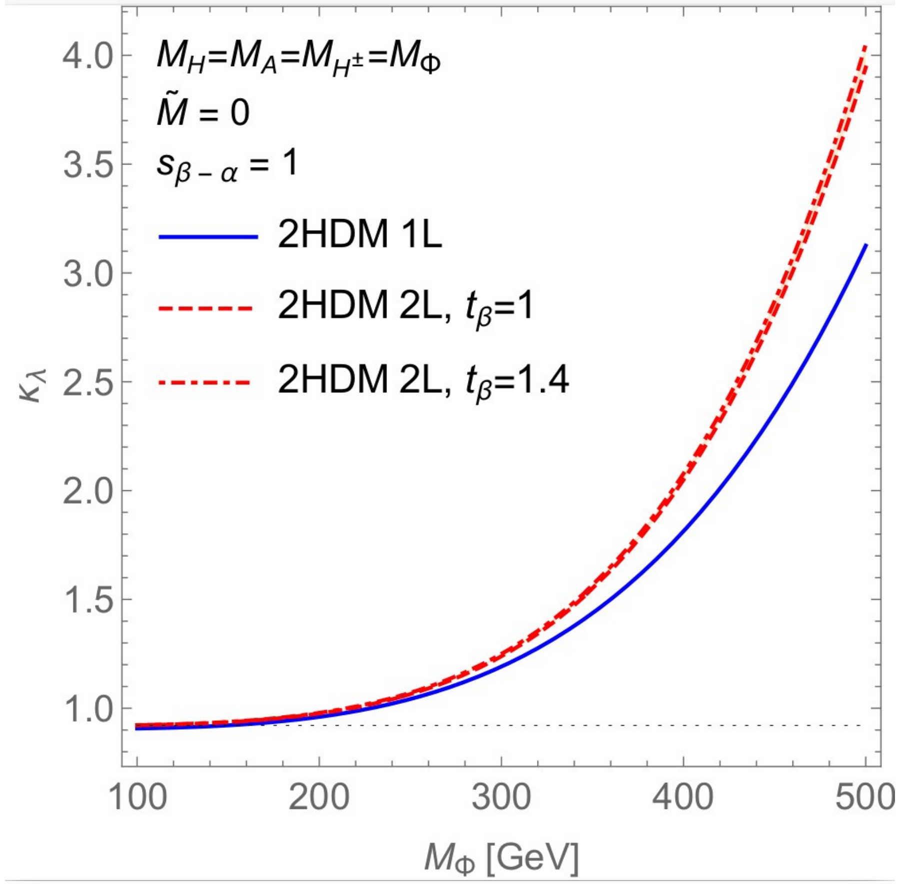


λ_{hhh} at two loops in more models

[JB, Kanemura '19]

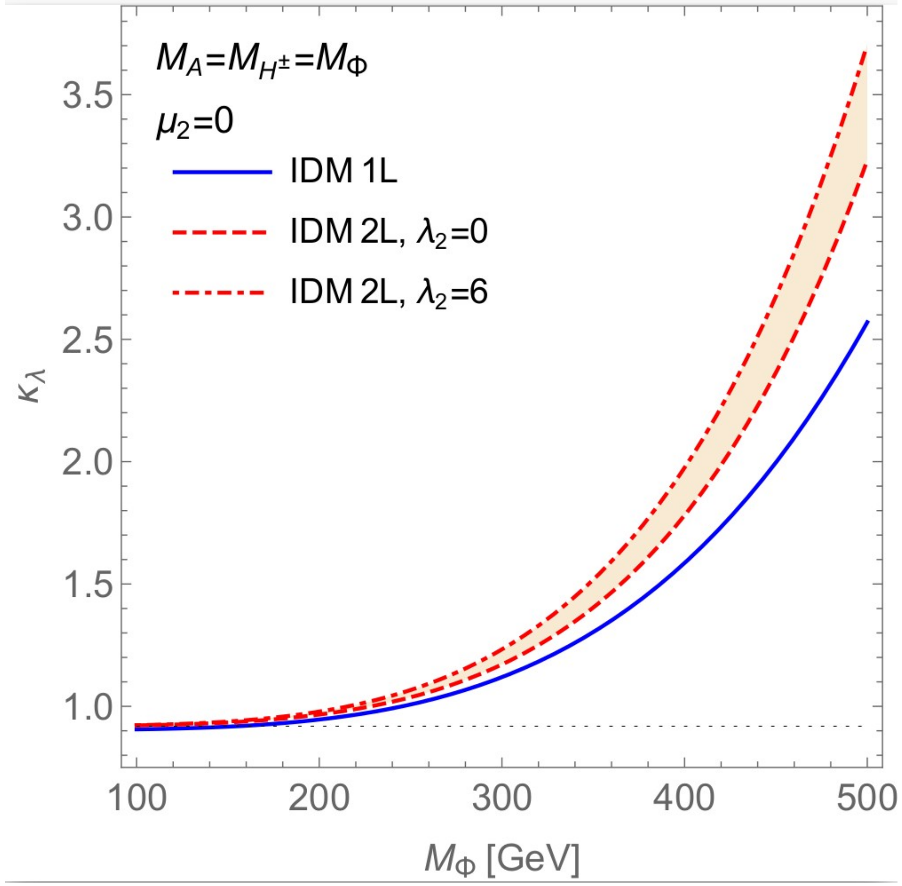
- Calculations in several other models: *Inert Doublet Model (IDM)*, *singlet extension of SM*
- Each model contains a **new parameter appearing from two loops**:

Aligned 2HDM → tanβ



tanβ constrained by perturbative unitarity → only small effects

IDM → λ_2 (quartic coupling of inert doublet)



λ_2 is less contrained → **enhancement is possible**
(but 2L effects remain <u>well smaller</u> than 1L ones)

Constraining BSM models with λ_{hhh}

i. *Can we apply the limits on κ_λ, extracted from experimental searches for di-Higgs production, for BSM models?*

ii. *Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?*

As a concrete example, we consider an aligned 2HDM

Based on

arXiv:2202.03453 **(Phys. Rev. Lett.) in collaboration with Henning Bahl and Georg Weiglein**

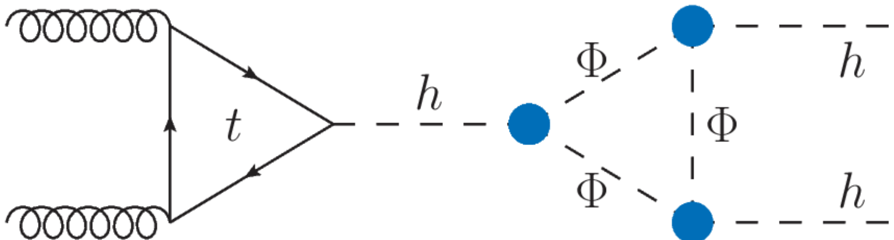
Can we apply di-Higgs results for the aligned 2HDM?

- Current strongest limits on κ_λ from ATLAS di-Higgs searches

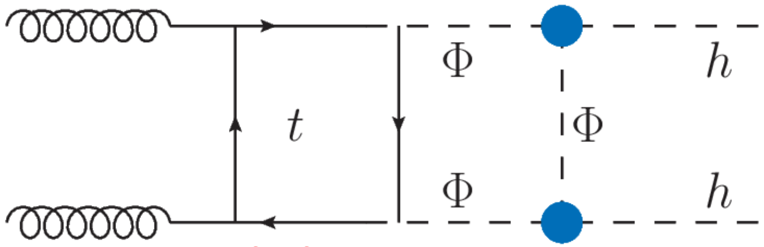
$-1.2 < \kappa_\lambda < 7.2$ [ATLAS-CONF-2024-006] [where $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$]

- What are the *assumptions* for the ATLAS limits?
  - All other Higgs couplings (to fermions, gauge bosons) are SM-like

    → this is **ensured by the alignment** ✓
  - The modification of λ_{hhh} is the only source of deviation of the *non-resonant Higgs-pair production cross section* from the SM



$\propto \mathcal{O}(y_t g^3_{hh\Phi\Phi})$ ***included***



$\propto \mathcal{O}(y_t^2 g^2_{hh\Phi\Phi})$ *not included*

→ We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

- **We can apply the ATLAS limits to our setting!**

A parameter scan in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

- Our strategy:
  1. **Scan BSM parameter space**, keeping only points passing various theoretical and experimental constraints (*see below*)
  2. Identify regions with **large BSM deviations in λ_{hhh}**
  3. Devise a **benchmark scenario** allowing large deviations and investigate impact of experimental limit on λ_{hhh}
- *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types, or other BSM models with extended Higgs sectors
- Constraints in our parameter scan:

experimental
  - 125-GeV Higgs measurements with `HiggsSignals`
  - Direct searches for BSM scalars with `HiggsBounds`
  - b-physics constraints, using results from [Gfitter group 1803.01853]

Checked with `ScannerS` [Mühlleitner et al. `2007.02985`]

  - EW precision observables, computed at two loops with `THDM_EWPOS` [Hessenberger, Hollik '16, '22]

theoretical
  - Vacuum stability
  - Boundedness-from-below of the potential

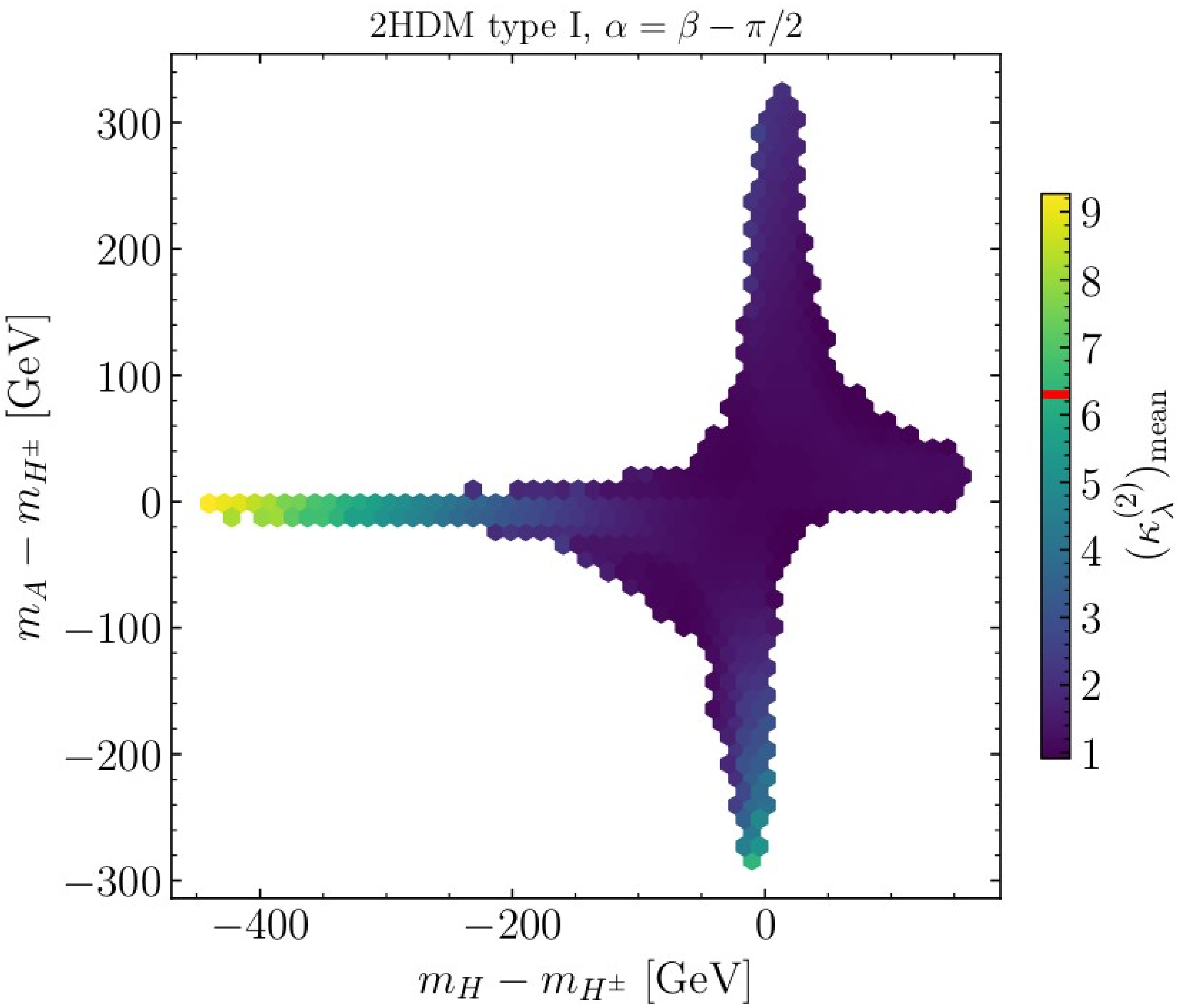
Checked with `ScannerS`

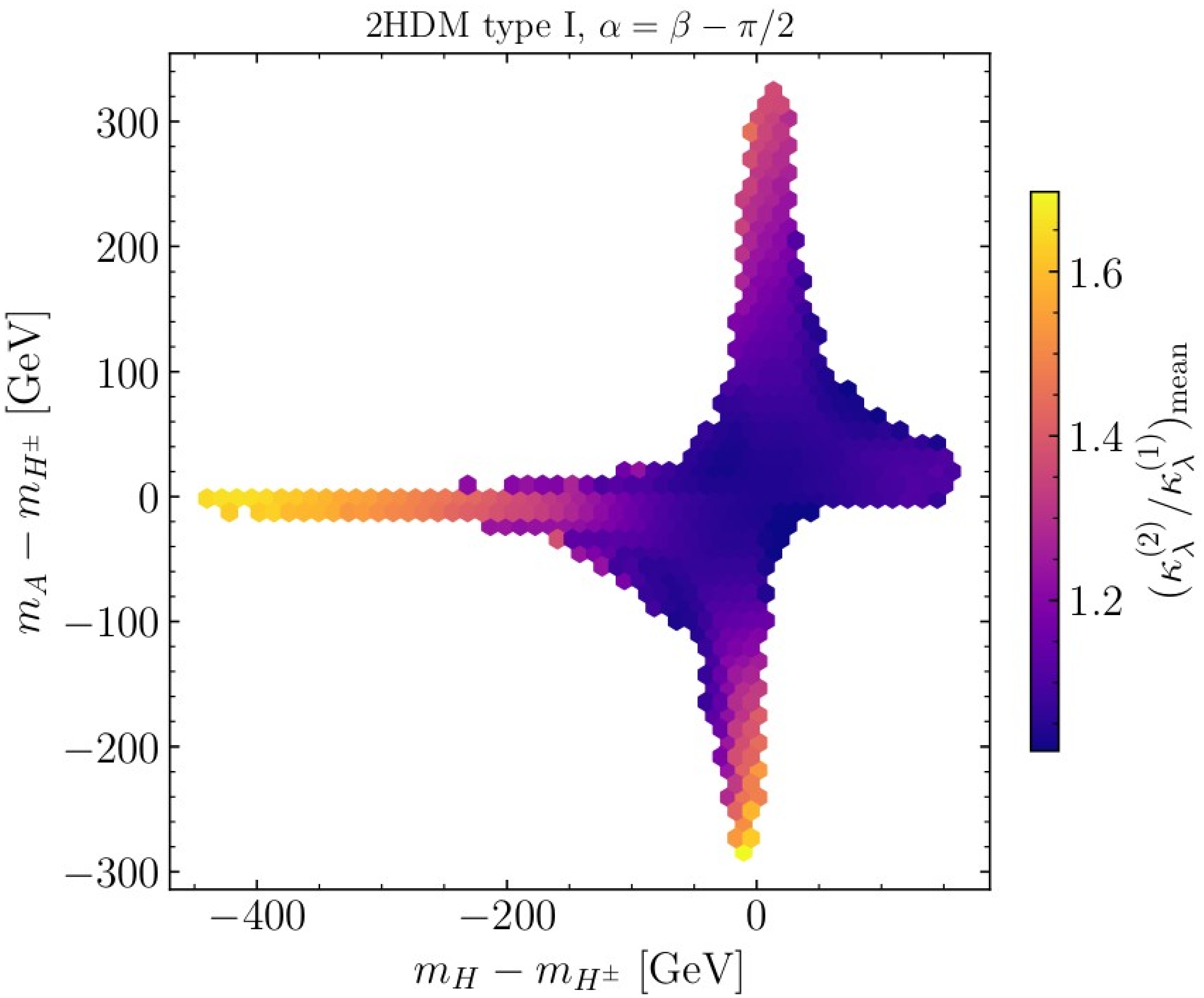
  - NLO perturbative unitarity, using results from [Grinstein et al. `1512.04567`], [Cacchio et al. `1609.01290`]
- For points passing these constraints, we **compute κ_λ at 1L and 2L**, using results from [JB, Kanemura '19]

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM} / (\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM} / (\lambda_{hhh}^{(1)})^{2HDM}$ [right] in ($m_H - m_{H\pm}$, $m_A - m_{H\pm}$) plane



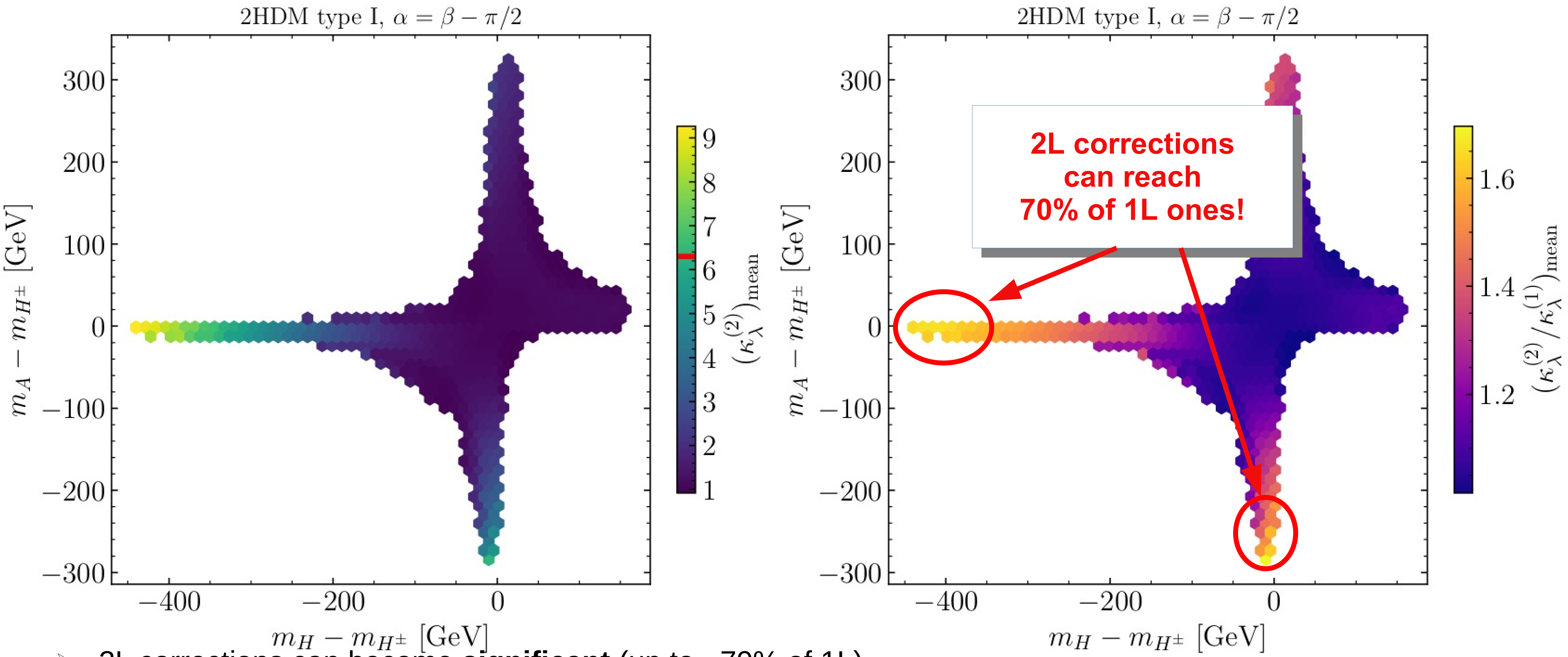


NB: all previously mentioned constraints are fulfilled by the points shown here

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane

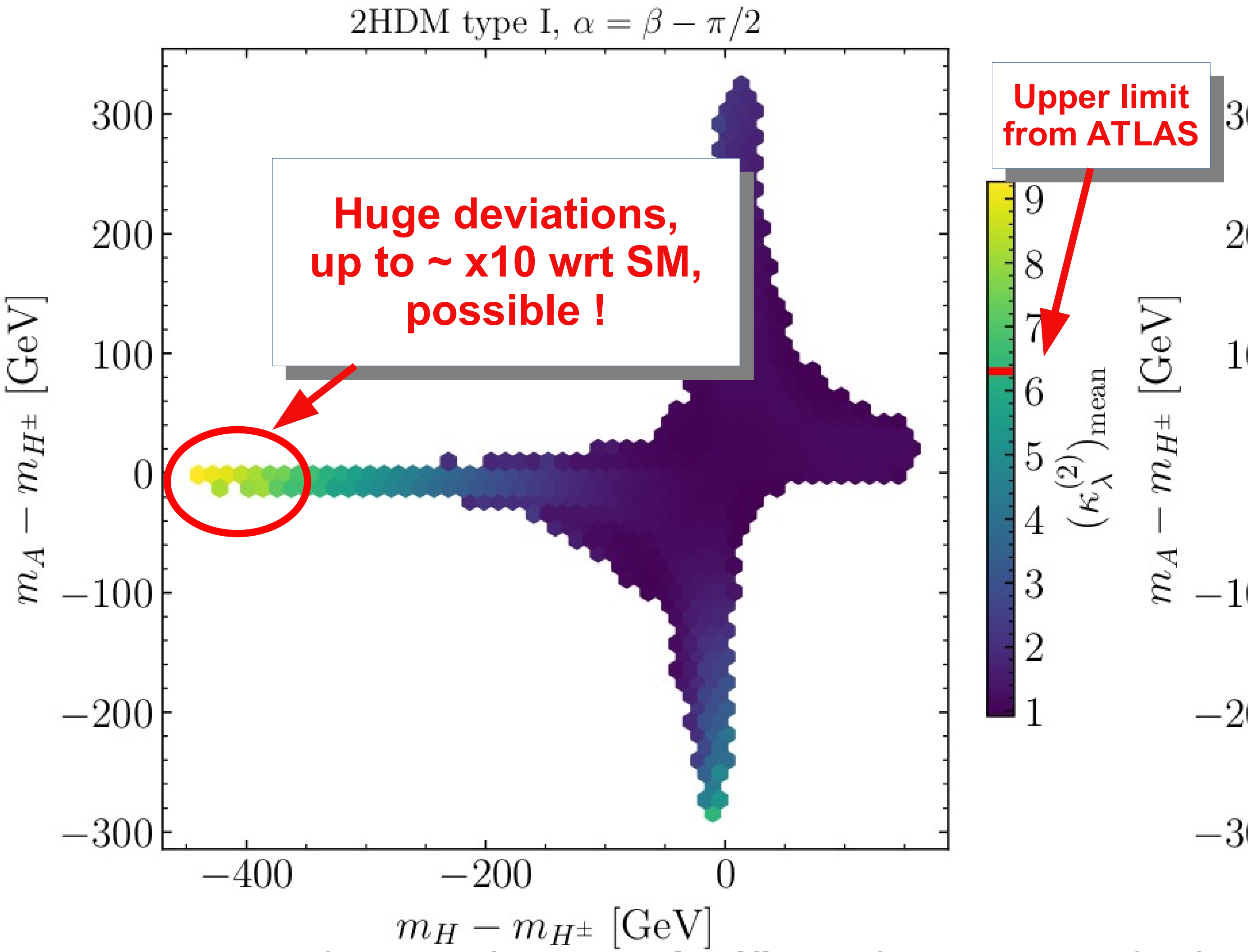


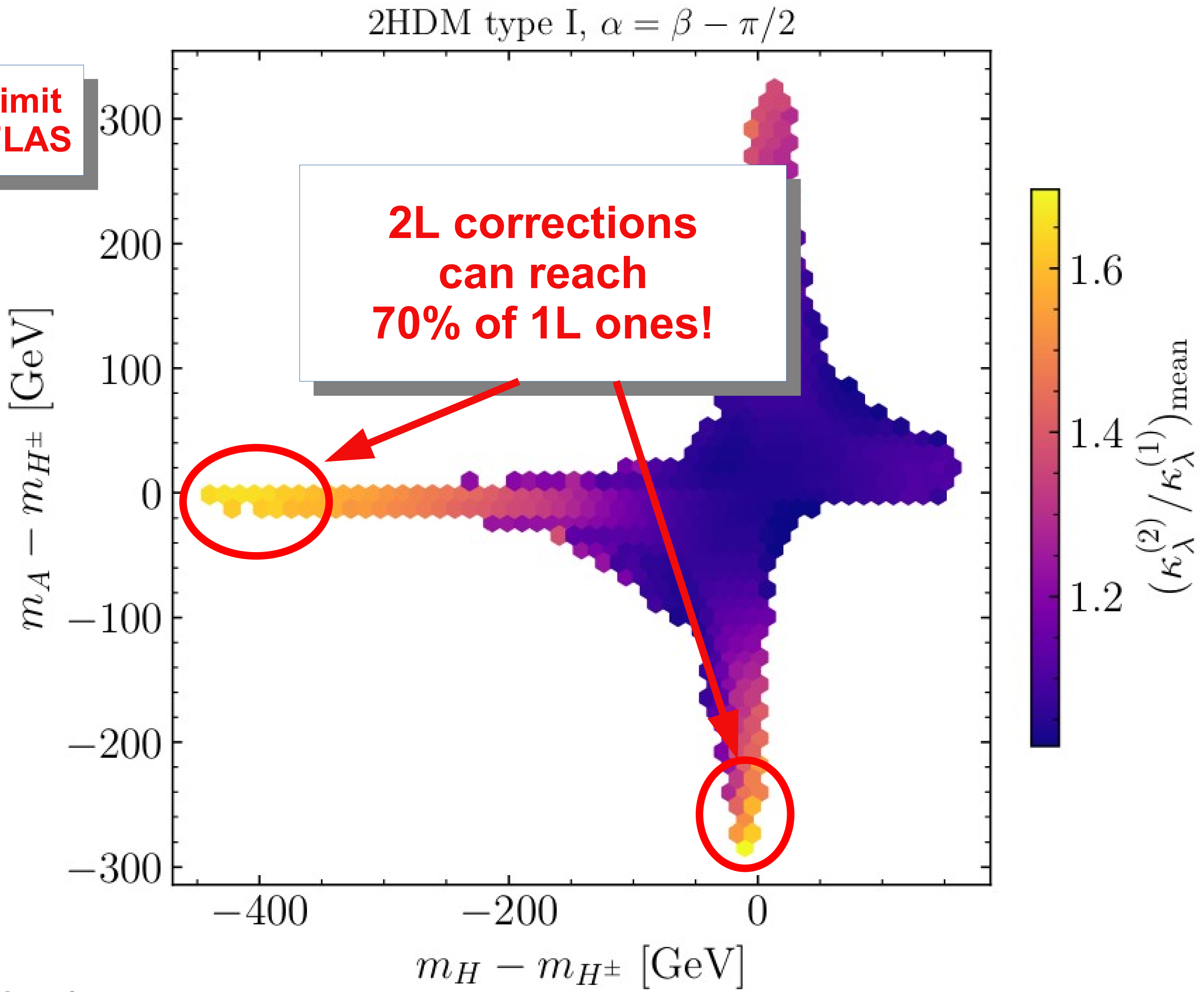
- 2L corrections can become **significant** (up to ~70% of 1L)

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane





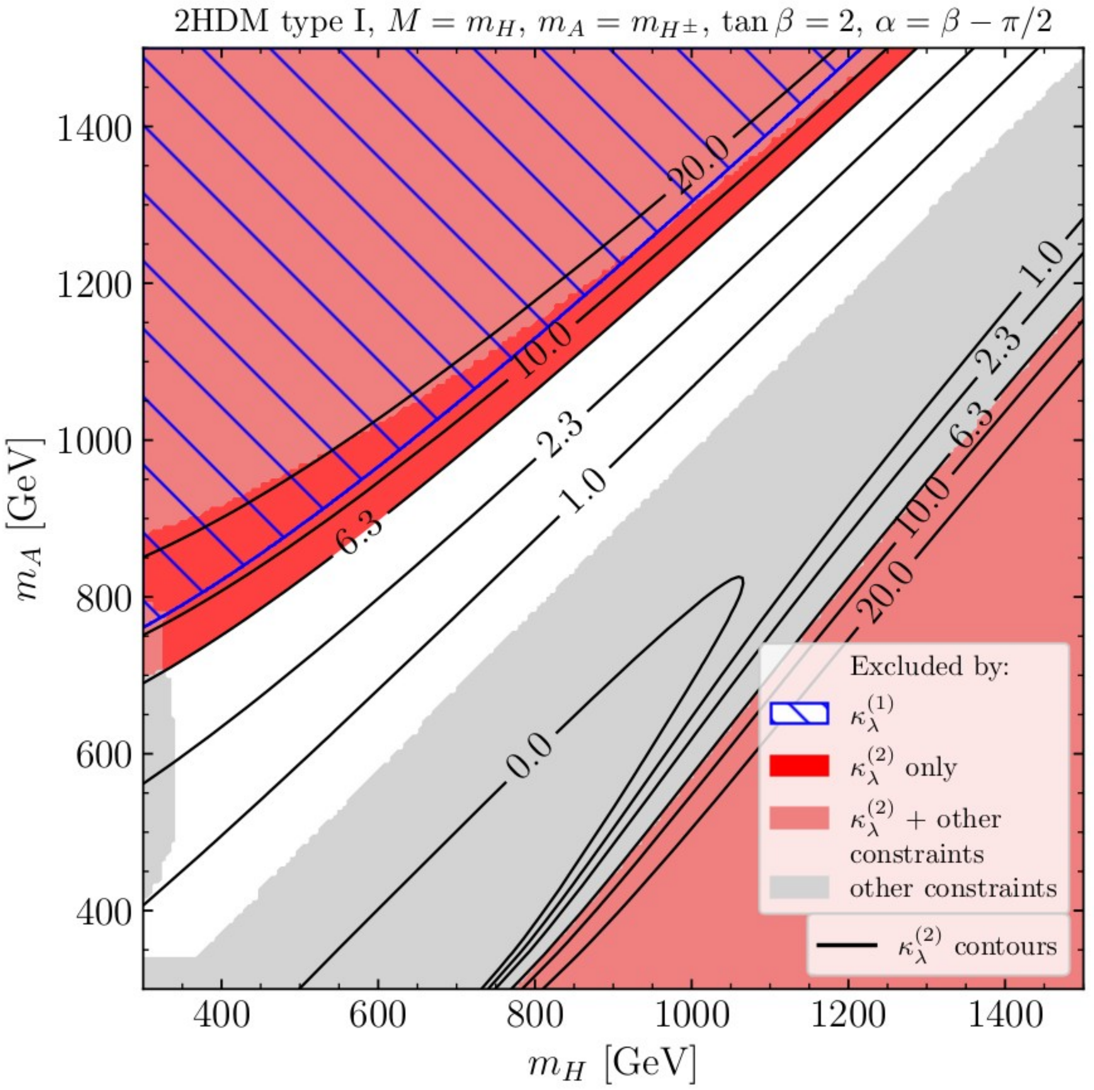
- 2L corrections can become **significant** (up to ~70% of 1L)
- **Huge enhancements** (by a factor ~10) of λ_{hhh} possible for $m_A \sim m_{H\pm}$ and $m_H \sim M$

A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take m_A=$m_{H\pm}$, M=m_H, tanβ=2



- ***Grey area:*** area excluded by other constraints, in particular BSM Higgs searches, boundedness-from-below (BFB), perturbative unitarity
- ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)}$ > 6.3 [in region where $\kappa_\lambda^{(2)}$ < -0.4 the calculation isn't reliable]
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)}$ > 6.3**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)}$ > 6.3 → impact of including 2L corrections is significant!

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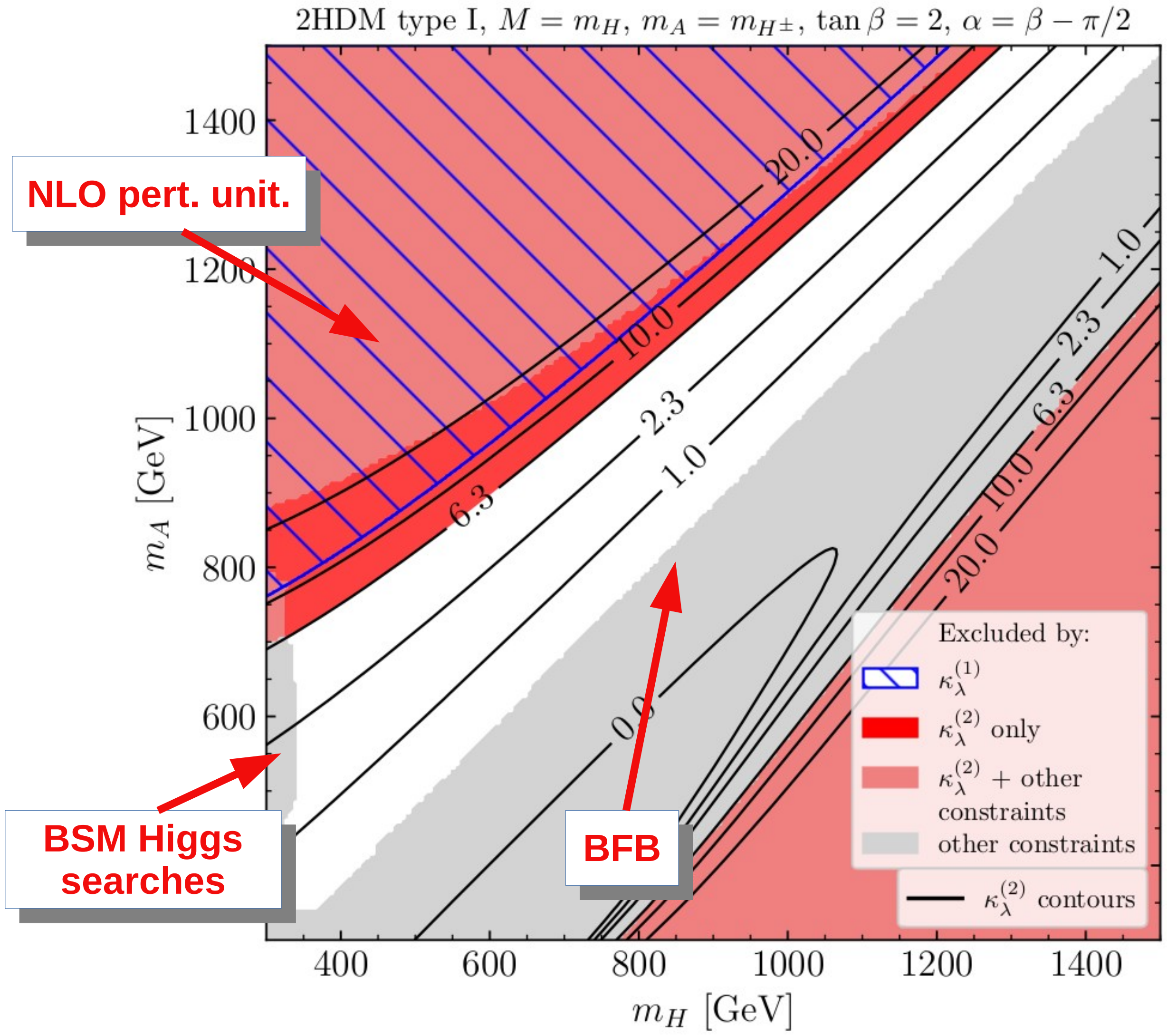
We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$



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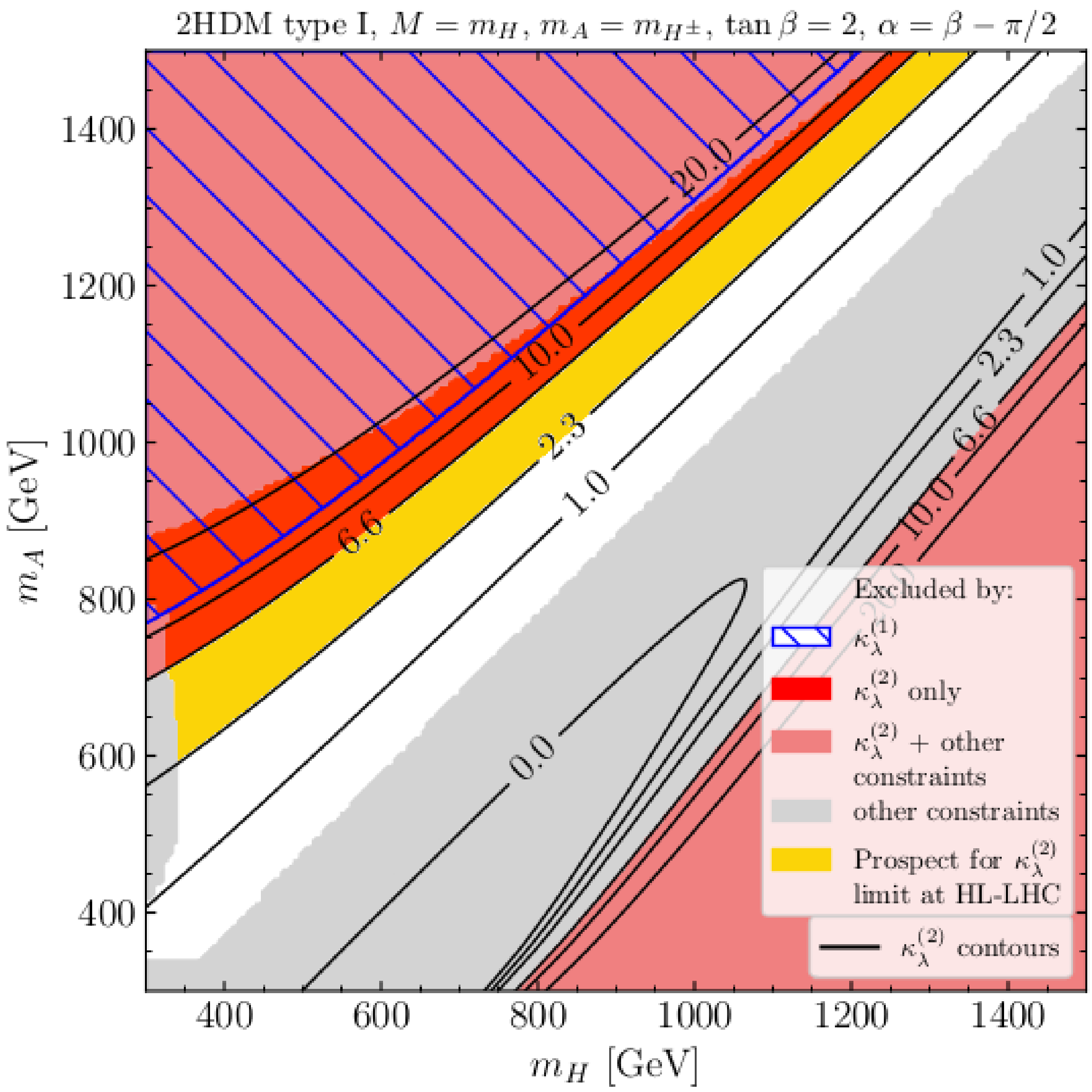
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$



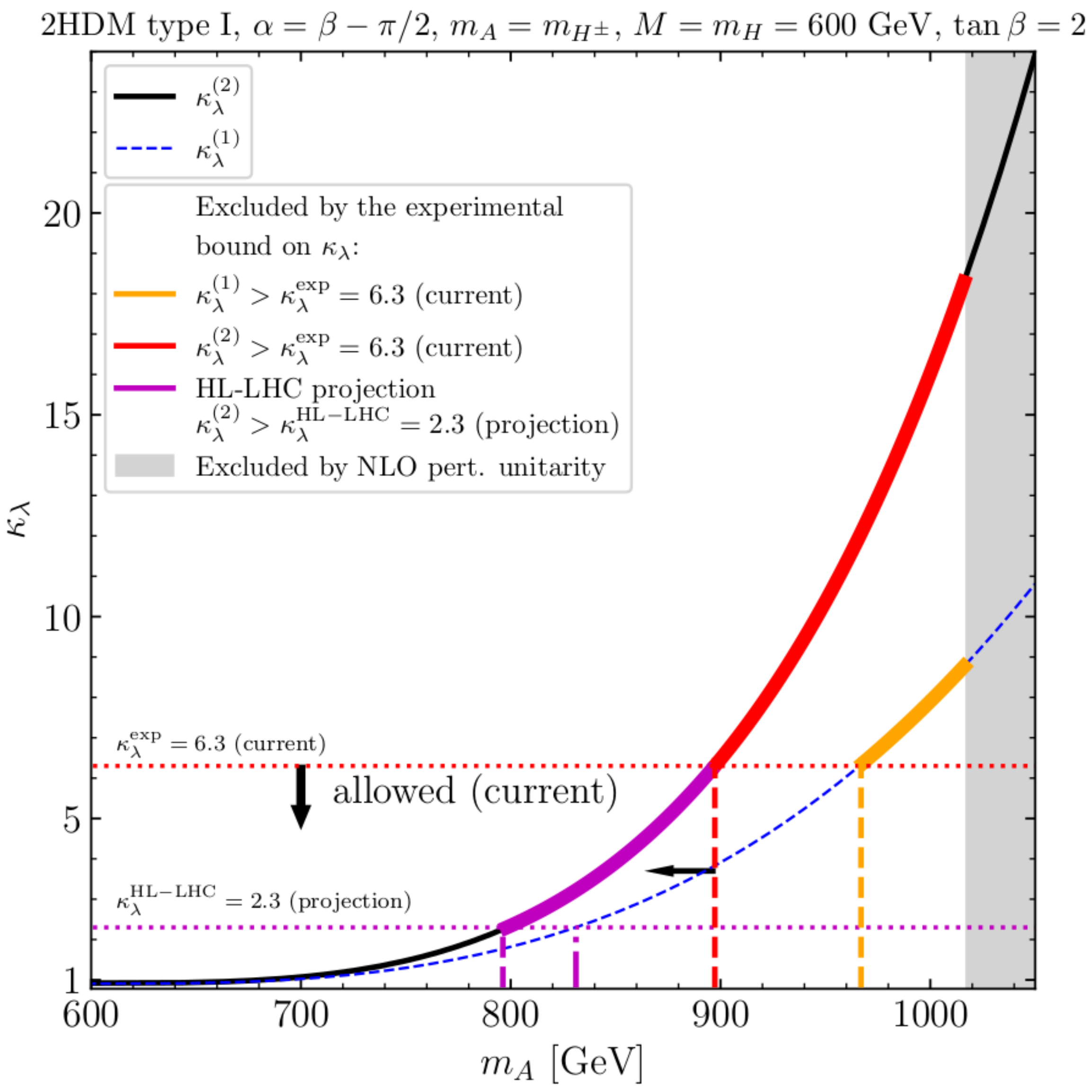
[Bahl, JB, Weiglein '23]

- ***Golden area:*** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)
- Of course, **prospects even better with an e^+e^- collider!**
- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

A benchmark scenario in the aligned 2HDM – 1D scan

Within the previously shown plane, we fix $M=m_H=600$ GeV, and vary $m_A=m_{H\pm}$

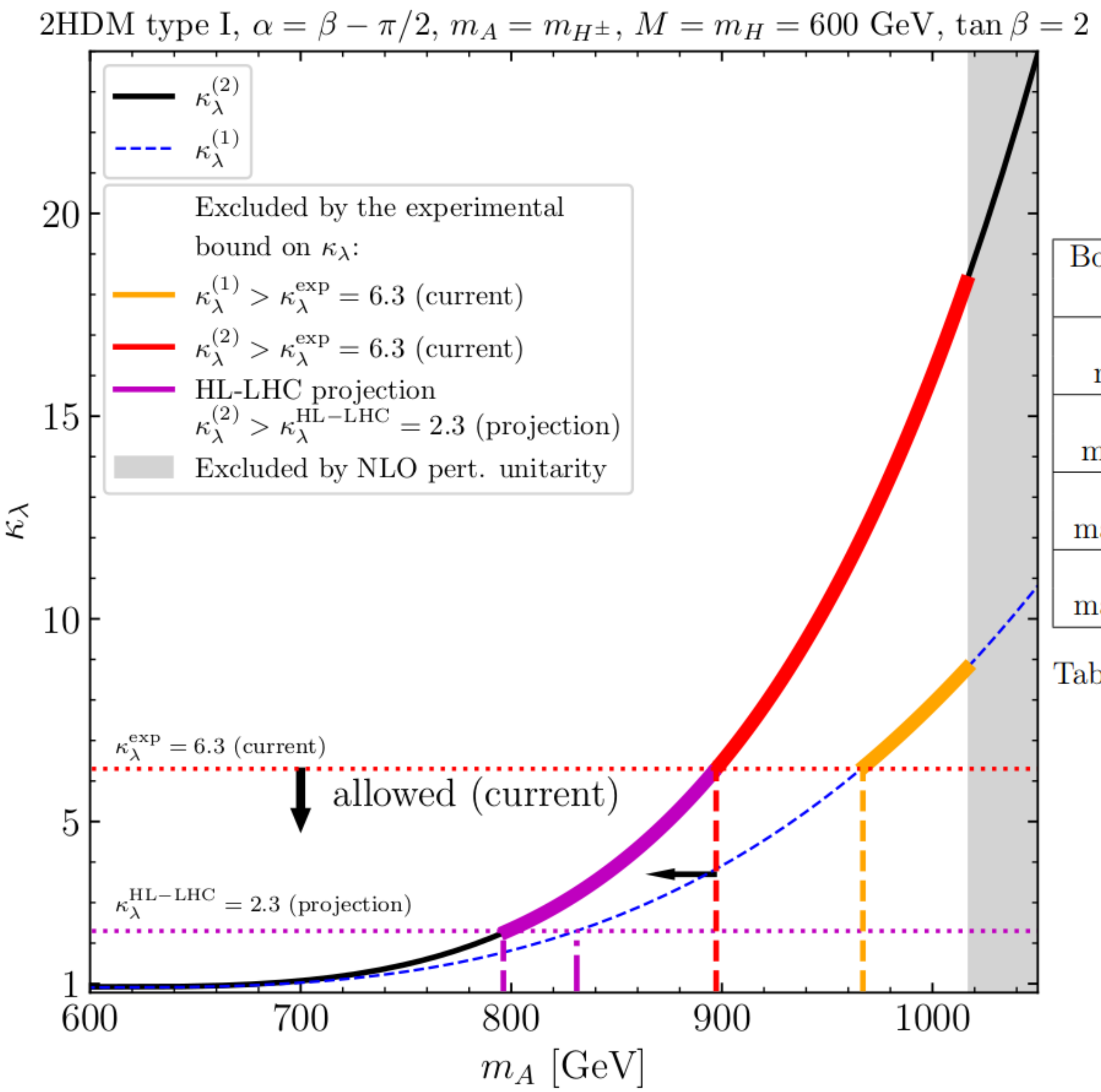
[Bahl, JB, Weiglein PRL '22]



- Illustrates the significantly improved reach of the experimental limit when including **2L corrections** in calculation of κ_λ
- A stricter choice for the perturbative unitarity constraint (grey) does not significantly change the region excluded by $\kappa_\lambda^{(2)}$

A benchmark scenario in the aligned 2HDM – 1D scan

[Bahl, JB, Weiglein PRL ‘22]

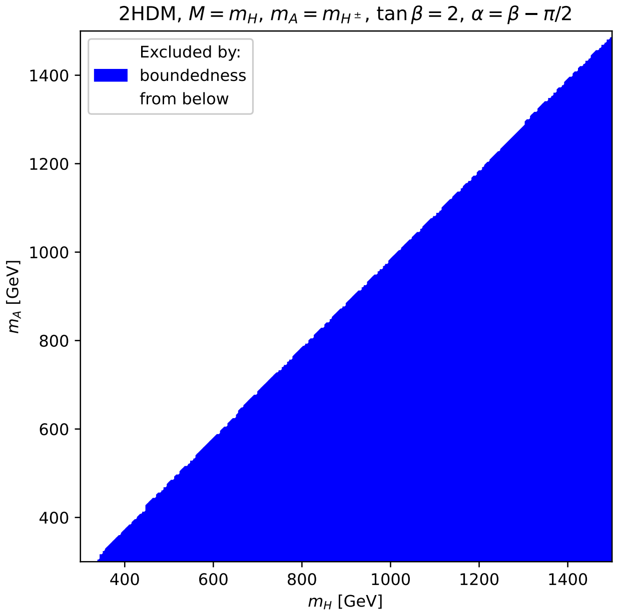


Bound on eigenvalues	$\max(m_A)$ with LO pert. unit.	$\max(m_A)$ with NLO pert. unit.	$\max(m_A)$ with with finite $\sqrt{s} \in [3\text{ TeV}, 10\text{ TeV}]$
$\max(\lvert a_i \rvert) < 1$	1161 GeV	1017 GeV	–
$\max(\lvert \mathfrak{Re}(a_i) \rvert) < 1$	1161 GeV	1033 GeV	1260 GeV
$\max(\lvert a_i \rvert) < 0.5$	917 GeV	937 GeV	–
$\max(\lvert \mathfrak{Re}(a_i) \rvert) < 0.5$	917 GeV	958 GeV	929 GeV
$\max(\lvert a_i \rvert) < 0.49$	911 GeV	933 GeV	–
$\max(\lvert \mathfrak{Re}(a_i) \rvert) < 0.49$	911 GeV	956 GeV	922 GeV
$\max(\lvert a_i \rvert) < 0.45$	889 GeV	912 GeV	–
$\max(\lvert \mathfrak{Re}(a_i) \rvert) < 0.45$	889 GeV	948 GeV	897 GeV

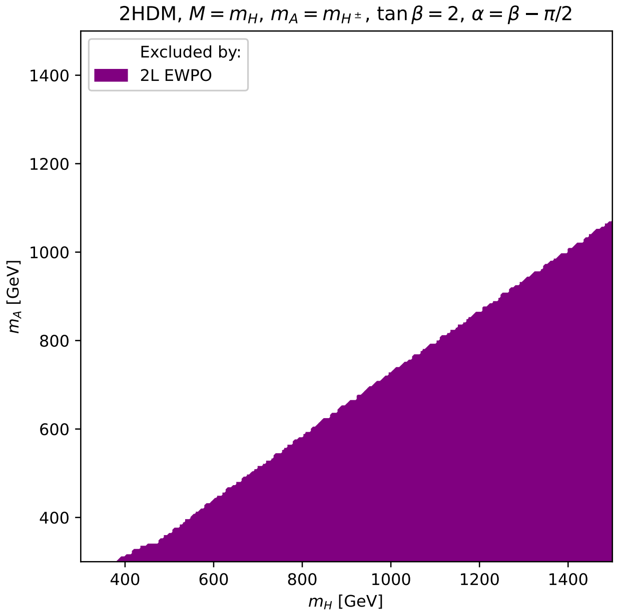
Table 1: Maximal values of m_A allowed in the benchmark scenario under the constraint of perturbative unitarity, at LO and NLO, and for different upper bounds on the $2 \to 2$ scattering eigenvalues used in the perturbative unitarity constraint. Note that tree-level scattering eigenvalues are all real, so there is no difference between using max or $\mathfrak{Re}$(max) for the left column.

2HDM benchmark plane – individual theoretical constraints

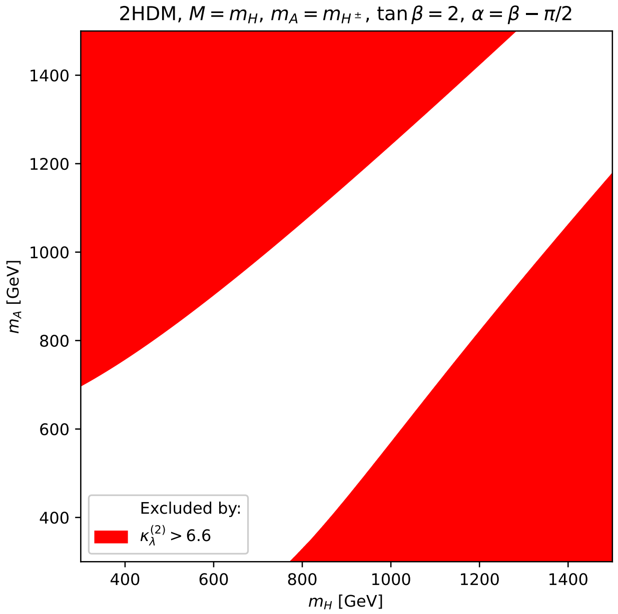
Constraints shown below are independent of 2HDM type



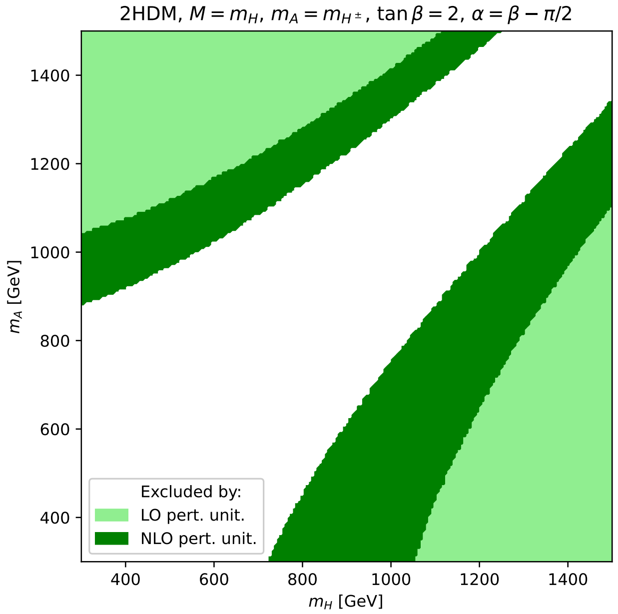
Boundedness from below



EW precision observables computed at 2L



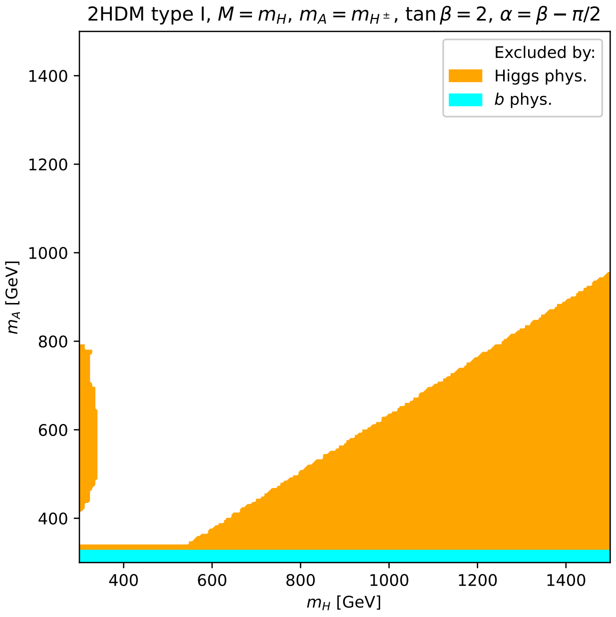
$\kappa_\lambda^{(2)} > 6.6$



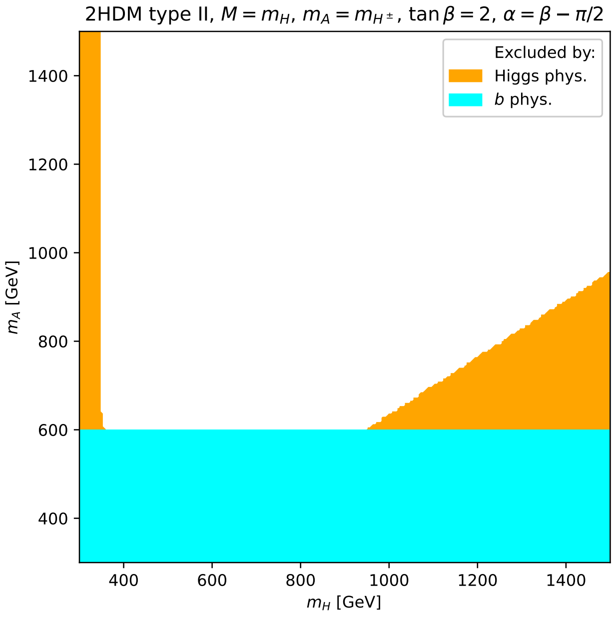
Perturbative unitarity at (N)LO

2HDM benchmark plane – experimental constraints

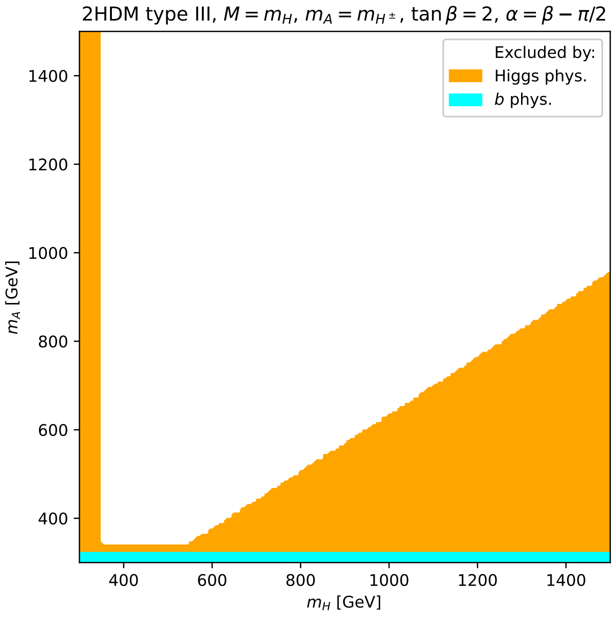
i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])



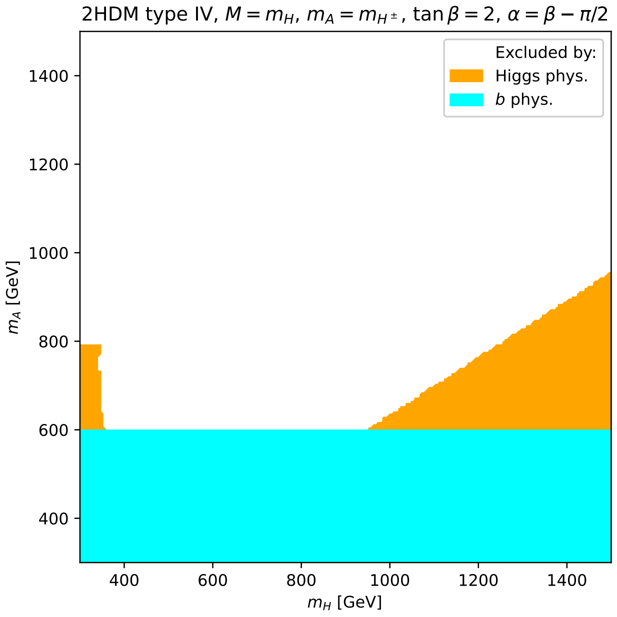
Type-I



Type-II



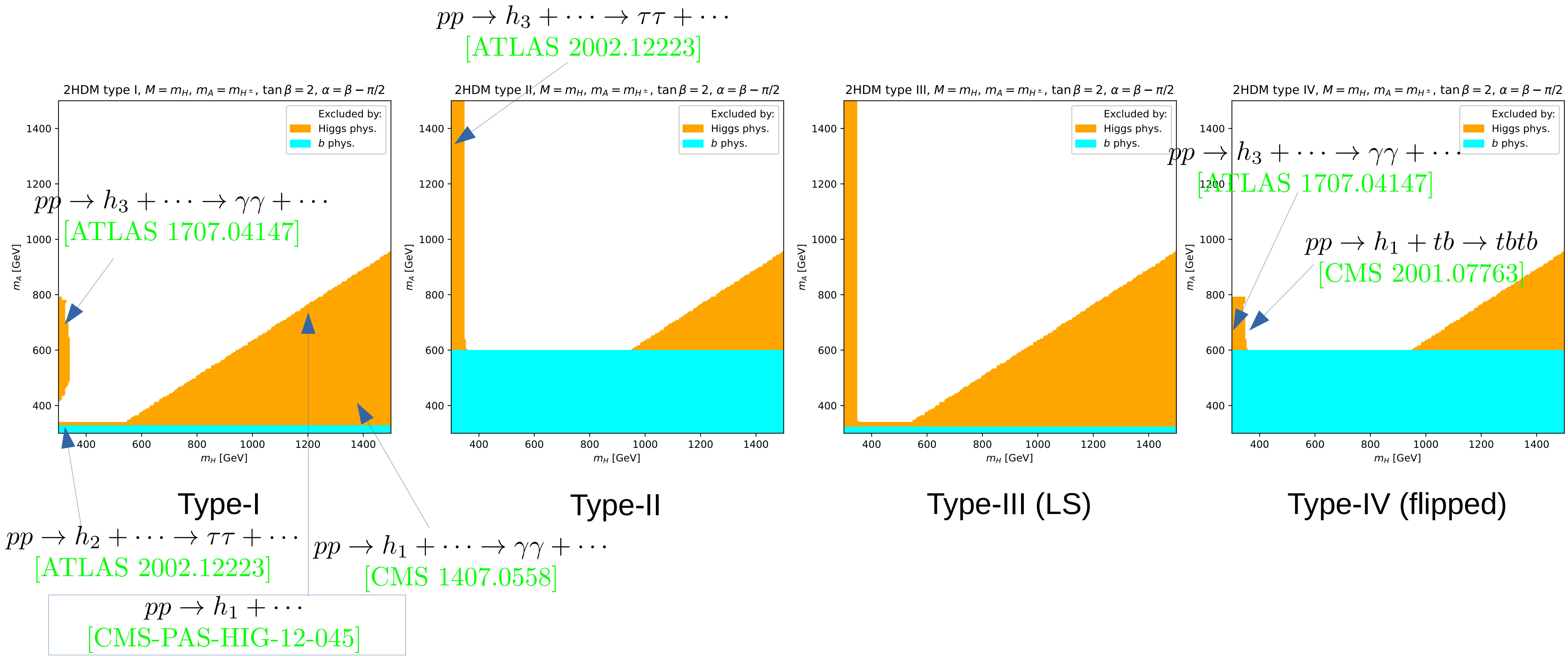
Type-III (LS)



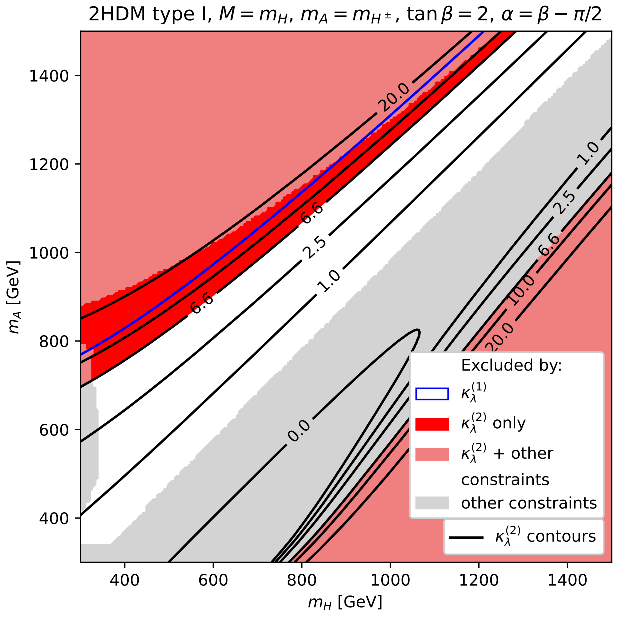
Type-IV (flipped)

2HDM benchmark plane – experimental constraints

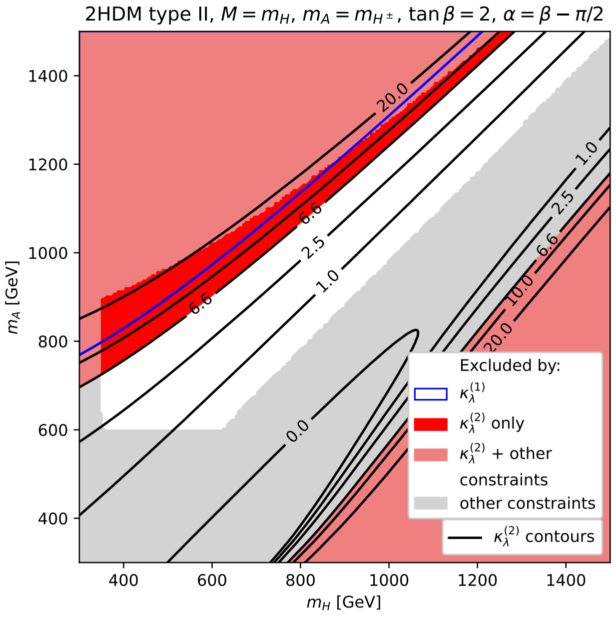
i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])



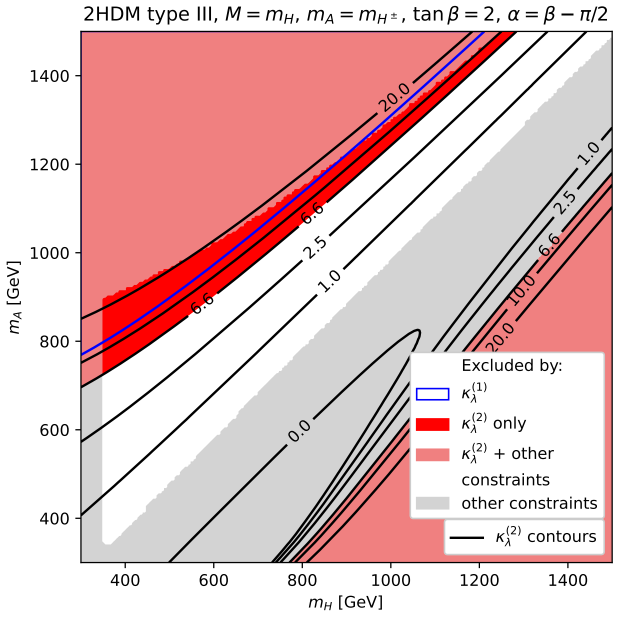
2HDM benchmark plane – results for all types



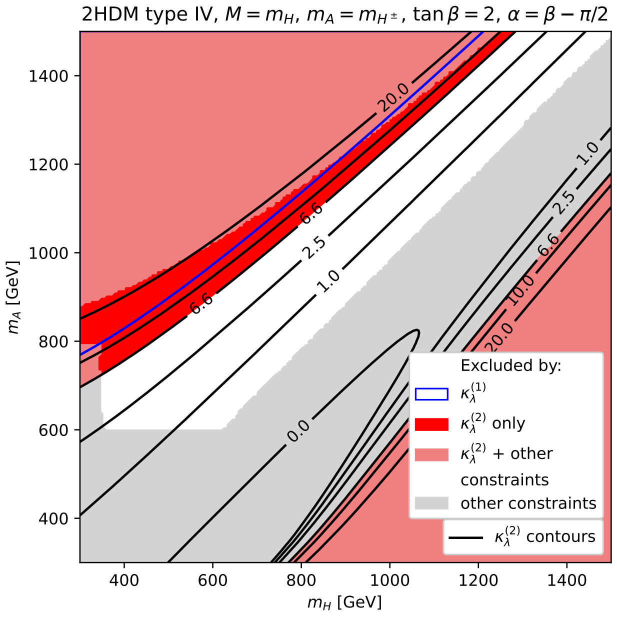
Type-I



Type-II



Type-III (LS)



Type-IV (flipped)

anyH3: full 1L calculation of λ_{hhh} in any renormalisable model